Student name:
Student number:

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University of Toronto

ECE 431F, (DSP)

MIDTERM TEST

October 18, 2000, 6:15PM to 7:45PM

Time allowed: 90 minutes. Closed book. 3 problems (9+9+7 = 25points)

Note: Please write your answers in the allocated space only. All answers should be properly explained by providing appropriate detailed arguments, reasoning, formulas, diagrams, etc..

Problem 1 (3+3+3=total 9 points)

A street has four kinds of traffic, comprised of:

- type 0 traffic that is not moving (e.g. parked cars, etc.)
- type 1 traffic that is always moving at exactly 10 meters per second (36km/h);
- type 2 traffic that is always moving at exactly 20 meters per second (72km/h);
- type 3 traffic that is always moving at exactly 30 meters per second (108km/h).

An ideal traffic control radar illuminates the roadway with a high frequency signal for speed measurement. For simplicity, assume that any traffic that is moving moves in the same direction toward the radar, and that a return signal, which you have been asked to analyze, has some or all of only the following exact frequencies present:

- a 30,000,000,000 Hz (30 gigaHertz) return, $g_0(t) = \exp(2\pi i 30000000000t)$, from type 0 traffic;
- a 30,000,001,000 Hz return, $g_1(t) = \exp(2\pi i 30000001000t)$ from type 1 traffic;
- a 30,000,002,000 Hz return, $g_2(t) = \exp(2\pi i 30000002000t)$, from type 2 traffic;
- a 30,000,003,000 Hz return, $g_3(t) = \exp(2\pi i 30000003000t)$, from type 3 traffic.

(a) (3points) The Department of Transportation asks you to purchase a sampler from a catalog of ideal data capture samplers, with sampling frequencies available from 1 kHz (1,000 Hz) to 100 gigaHertz (100,000,000,000 Hz), available in 1 kHz steps. Since there is increasing cost for increasing frequency, you are asked to propose the minimum such sampling frequency, F_s, required in order to avoid irrecoverable aliasing in the return signal, $g(t) = c_0 g_0(t) + c_1 g_1(t) + c_2 g_2(t) + c_3 g_3(t)$ for arbitrary constants c_0, c_1, c_2, and c_3. Explain your reasoning for this choice.

(b) (3points) Assume that you have a sequence of numbers $g[n] = g(nT_s)$, where $T_s = 1/F_s$ (where F_s is that which you selected above). Propose a method of discrete time signal processing in order to determine the constants c_0, c_1, c_2, and c_3 present in the original signal $g(t)$, by way of discrete time processing of $g[n]$. Explain your method.

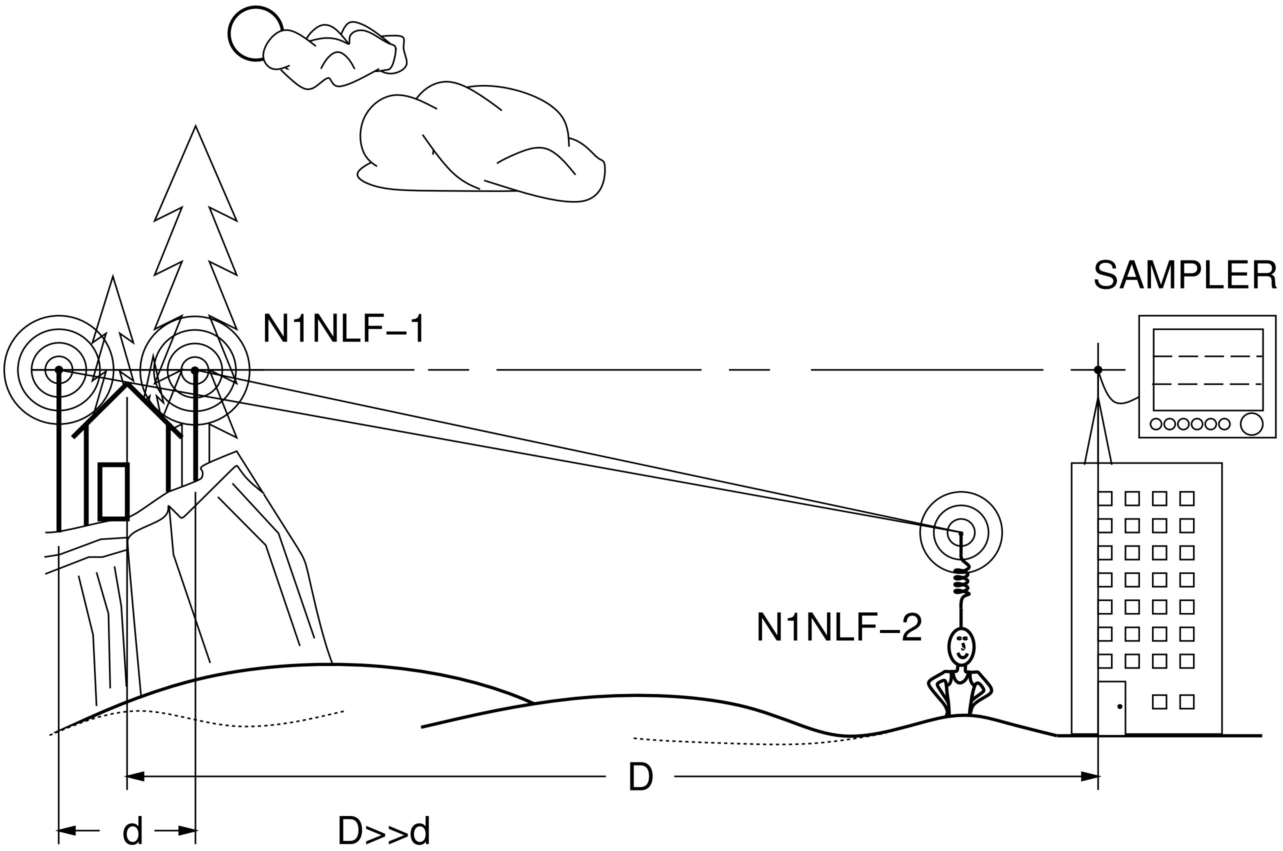
(c) (3points) Assuming the minimum sampling frequency you chose for F_s, what is the minimum number of uniformly spaced samples of $g(t)$ required to determine the constants c_0, c_1, c_2, and c_3. Explain your reasoning.

Problem 2 (3+3+3=total 9 points)

Ham radio station N1NLF-1 is capable of operating in four possible modes, transmitting at:

- exactly 7.5 MHz (7,500,000 cycles per second), wavelength = 40m;
- exactly 3.75 MHz (wavelength 80m);
- exactly 1.875 MHz (wavelength 160m);
- any fixed frequency in the frequency range from 440 MHz to 450 MHz.

Two ideal point source antennas, located $d = 40m$ apart are directly in line of sight of a listening post and monitoring station equipped with a sampler (SAMPLER) on the rooftop of a building as shown in the figure below.



Assume that the same signal is fed to both antennas, and that the signal received from each antenna has equal strength (e.g. $D >> d$), and that linearity and superposition applies to the received signal from each.

The sampler coherently operates at one fixed frequency of 10 megasamples (10,000,000 samples) per second, and can acquire four complex–valued samples at a time.

It is known that station N1NLF-1 always transmits at the same frequency to a mobile station N1NLF-2 located at ground level.

The following four samples are measured at the listening post:

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   1.0000            -0.7071+ 0.7071i  -0.0000- 1.0000i   0.7071+ 0.7071i
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where $i = \sqrt{-1}$. The four samples are known to be exactly 1, $(-1+i)/\sqrt{2}$, $-i$, and $(1+i)/\sqrt{2}$. For simplicity, assume that no other interference, reflections, noise, or other imperfections exist in the system.

(a) (3points) Compute the four point DFT of the four samples given. Show your work and calculations in detail.

(b) (3points) Compute the signal strength (e.g. magnitude squared) at each of the four values found in (a). Explain the intuitive physical significance of this set of four signal strengths. From these signal strengths, together with your knowledge of the situation, propose an hypothesis for what frequency or frequencies station N1NLF-1 could be transmitting at.

(c) (3points) You are asked to present your findings at a board meeting, and would like to give a plausible explanation for a value midway between the two highest values you found in (b). Propose a method for determining a value midway between these two highest values, and explain why your method makes intuitive sense, within the context of discrete time signal processing. Use your method to calculate the value midway between the two highest values you found in (b).

Problem 3 (7 points)

(scan and insert problem 4.60 from the course textbook here)