

Understanding what a camera measures

Abstract

A great deal of mathematical processing of pictures has been developed, to create the very active field of image processing, but with little or no regard to what the numbers coming out of a camera actually mean. The purpose of this paper is to ask, and attempt to answer, the fundamental question “what does a camera measure”. Once we understand what a camera measures, we can introduce a new kind of image processing that works in *lightspace* rather than in *image space*.

1 What does a camera measure

Cameras are often present in computer vision systems that are used to measure various quantities [1], yet an often overlooked question is “what does a camera itself measure”?

A camera actually acts as an array of sensors that each measure light. In an electronic camera these sensors are typically the outputs from the pixels.

Let us construct (either as a thought experiment, or as a do experiment that can actually be carried out) a simple one pixel camera. Once we understand what one pixel measures, we can then better understand what can be measured from many pixels working together.

Perhaps the simplest way to construct our one pixel camera is to use a light sensor. The cheapest and most common kinds of light sensors are usually photoresistors, such as the cadmium sulphide photocells found in the small controllers that automatically turn street-lights on after dark.

Cameras measure electromagnetic energy, but unlike an ideal radio receiving antenna that one might connect to a spectrum analyzer, a camera measures electromagnetic radiation only within a certain region of the electromagnetic spectrum. Typically cameras are sensitive to electromagnetic energy falling within or close to the visible, portion of the spectrum and a camera’s sensitivity to light is not very flat across the spectrum over which it is sensitive. Thus a camera is certainly not a radiometer, and the measurements that it makes are certainly not radiometric. Cameras have spectral sensitivities that are different from that of

the human eye, so cameras are not photometers either. But the output from a camera can still be quantified irrespective of its unique spectral response function, so cameras are referred to as quantimetric devices [2].

Although cameras do not output a linear measure of light, with the right computations, we can derive such a linear measurement, so that the camera itself can be used as a measurement instrument. The quantimetric unit, typically denoted by the letter q , is usually made relative to some reference value, q_0 , so that it can be expressed as a ratio, or as a logarithmic ratio (often in decibels).

In this simple experiment to create and understand a one-pixel camera we will consider quantimetric analysis of the Radio Shack Cadmium Sulphide photocell because it is readily available as a test sample (Radio Shack part number 276-1657). However any surplus photocell, or even a real camera, may be used.

1.0.1 Construction of a one pixel camera

Photoresistors are devices in which resistance is a function of incident light. Typically an increase in light results in a decrease in resistance. Most are known to obey an empirical law:

$$R = R_0 q^{-\gamma}, \quad (1)$$

where γ is usually a positive constant less than one. The fact that γ is less than one indicates that the photocell tends to compress dynamic range. Cameras also compress dynamic range in a similar way. The Radio Shack 276-1657 CDS cell has $\gamma = 0.75$, allowing us to calculate the ratio of resistance change for a change in the quantity of light. (See Chapter 3 of the University of Toronto Physical Sciences PSCB01H3S course text, <http://www.utoronto.ca/quick/PSCB01H3S/Manual/>).

Let us assume that a camera is an instrument for converting light into numbers. We will thus construct a camera from the photocell by simply connecting it to an ohm meter. A satisfactory ohm meter might be the MICRONTA 22-201U which has a moving coil meter movement, as shown in Fig. 1.

The needle swings to the right when the resistance is less. Like most similar ohm meters, ∞ is at the far

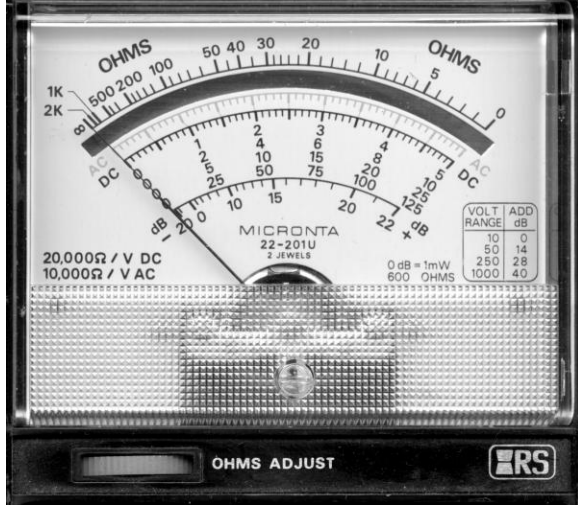


Figure 1: Typical resistance meter used in photocell experiment.

left and 0 is at the far right, with intermediate values in between.

It is more convenient to consider conductance, which is the reciprocal of resistance, and to define the camera's response function, f , as conductance of the photocell. Equation (1) now becomes:

$$f(q) = \beta q^\gamma, \quad (2)$$

where $\beta = 1/R_0$. The quantimetric function, f , increases with increasing quantity of light, q . Thus the needle on the meter will be at the far left when $f = 0 = q$, and will swing more to the right as f and q increase.

We wish to determine f as a function of q (e.g. suppose that we did not already know the relationship between f and q). Suppose that we have just a lamp and the camera (photocell plus ohm meter), together with a piece of black cardboard we can use to cover half of the lamp, but that we have no other measurement instruments. We assume that the lamp is round or has some other shape that makes it easy to cover up exactly half of it.

We are therefore able to cover half of the lamp, and then point the lamp at the camera (sensor), and take a reading $f(q_0)$, (See Fig 2(a)), and then uncover the lamp while leaving it exactly in the same place. Uncovering the lamp exactly doubles the quantity of light received by the camera (sensor), so that we then know what $f(2q_0)$ is. (See Fig 2(b).) Although we do not know the absolute quantity q_0 , we do know the relative quantity $2q_0/q_0 = 2$, e.g. we do know the fact that we have exactly doubled the quantity of light, and we can record a set of ordered pairs $(f(q_0), f(2q_0))$. We can put the lamp at various different distances from

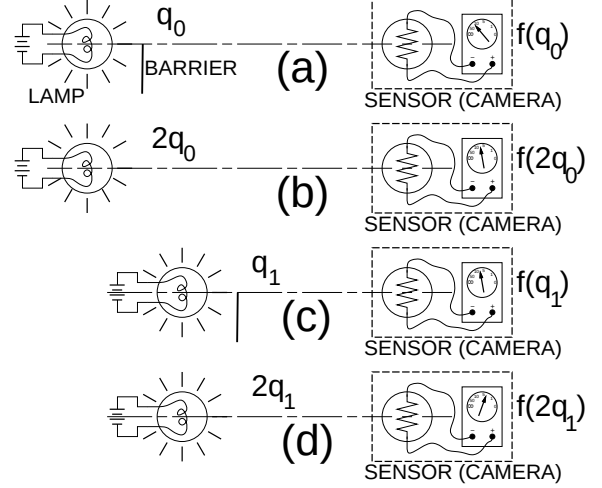


Figure 2: Photocell experiment: for each of various distances between the lamp and the camera, a reading is taken with the lamp exactly half covered, versus with the lamp not covered. Other than the camera, lamp, and black card (BARRIER), no other devices (such as rulers or other measurement instruments) are needed in order to characterize the response of the photocell to light.

the camera (sensor) and for each such lamp position, we can generate an ordered pair (see Fig 2(c) and (d)). Let us suppose that we take ten such ordered pairs of measurements, e.g. continuing with $(f(q_1), f(2q_1))$ and so on, all the way up to $(f(q_9), f(2q_9))$. We can plot these ordered pairs on a graph, as shown in Fig 3. This is just like an (x, y) plot, except that the axes are actually functions, $f(q)$, and $f(2q)$. The first axis is f so it is convenient to use the next letter of the alphabet, g , after f in order to denote the other axis. Thus we have an (f, g) plot — a plot of a function against a plot of a function of a function, where $g = f(2q)$.

We can fit a curve through the points. The solid line shows one possible curve, namely a straight line of slope approximately 1.68. The dotted line shows another possible curve. Repeated measurements, however, lead us to believe that the relationship is simply $g = 1.68f$, as shown by the solid line.

This is simply a plot of the photocell's response function $f(q)$ against a contracted (squashed in) version of the same function $f(2q)$. Such a plot is called a *comparametric plot*.

The notion of a parametric plot is familiar to almost everyone. A parametric plot of a circle, for example, is simply a set of ordered pairs $(r\cos(\theta), r\sin(\theta))$ where the parameter θ takes us counterclockwise around the circle. A *comparametric plot* is just a special kind of parametric plot, where both axes pertain to the same function but at different rates.

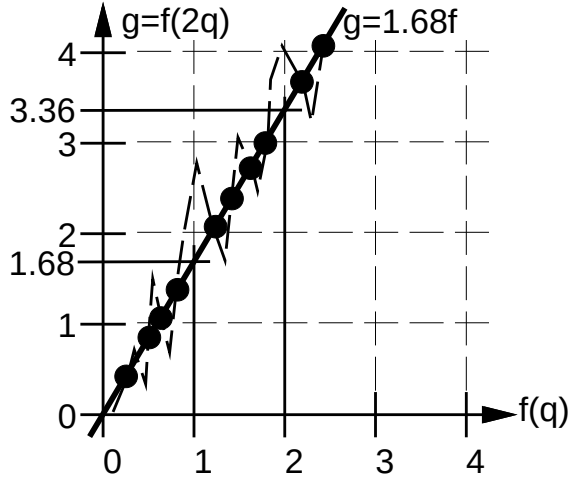


Figure 3: Datapoints from photocell experiment: ordered pairs of points taken by reading the camera with the lamp uncovered and exactly half covered.

1.0.2 Solving comparametric equations

From the comparametric plot shown in Fig 3 we have determined that $g = f(2q) = 1.68f(q)$. This equation, $g = 1.68f$ is called a comparametric equation. In general, solving a comparametric equation $g(f(q)) = f(kq)$, for some comparametric ratio k (in this case $k = 2$) means determining a family of possible functions $f(q)$ that satisfy this equation.

In our specific case, therefore, we wish to know what possible functions $f(q)$ give a straight line (of slope 1.68) when plotted against themselves contracted (by a factor of 2).

Fig 4 depicts two plots, one being a smooth function $f(q)$ who's comparametric plot is a straight line of slope 1.68, and the other being a quasi-periodic function who's comparametric plot is also a straight line of slope 1.68. Both of these functions have the same comparametric plot. Both are solutions to the comparametric equation $g = 1.68f$.

The quasi-periodic function illustrates that any function can be specified on, for example, the interval from q to $2q$, and then merely replicated into the interval to the right scaling by $(2*, g\circ)$, e.g. by stretching to twice the width and composing to $g()$ of the height, in this case, merely multiplying by 1.68 times the height, since in this specific case, g is linear. This recipe can be applied recursively in both the left and right directions. Therefore, in general, solutions to comparametric equations are not unique. However, we might choose a function that is semimonotonic, with semimonotonic slope, semimonotonic curvature, (and so-on, including possibly further derivatives), of which $f(q) = \beta q^\gamma$ is

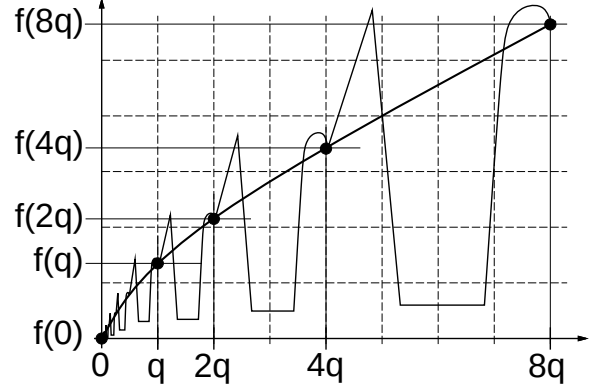


Figure 4: Two examples of possible solutions to the comparametric function $g = 1.68f$. One solution, plotted as the thick line, is given by $f(q) = q^{0.75}$. Another solution plotted as the thin line, consists of a triangular wave, square-shaped wave, and round shaped wave between q and $2q$ so that we can see how the waveform has comparametric periodicity. In each "period", the function is stretched by a factor of two as we go to the right (or contracted by a factor of two as we go to the left), and composed by a factor of $g()$ as we go to the right, or $g^{-1}()$ as we go to the left. This phenomenon is called comparametric periodicity. Thus there is a comparametric uncertainty in the solution to a comparametric equation.

the preferred general solution to $g = 2^\gamma f$.

1.1 Directly solving a comparametric equation by unrolling while collecting the data

We started by knowing the response function of the photocell, e.g. knowing the solution of the comparametric equation, and then confirming that this known function was in fact a solution.

Now suppose we did not know the solution, and did not know how to solve the comparametric equation, in general.

In this case, we can construct a simple way of simultaneously collecting comparametric data, and arriving at a numerical solution to the comparametric equation, namely, to obtain samples of the function $f(q)$.

Refer back to Fig 2(a) where half the lamp is blocked with the black cardboard, to obtain $f(q_0)$. Now suppose we unblock the lamp, as shown in Fig 2(b) to obtain $f(2q_0)$. The **important next step** is to now cover exactly half the lamp again and then move the lamp toward the photocell until the observed meter reading is exactly the same as it was when the lamp was not covered. Thus the situation as shown in Fig 2(c) will be that $q_1 = 2q_0$.

The procedure is repeated. Uncover the lamp to obtain $f(2q_1) = f(4q_0)$. Then cover it again, and move it still closer to the sensor, until the reading is the same as it was in Fig 2(d). Then uncover the lamp again, to obtain $f(4q_1) = f(8q_0)$.

Plotting these data points will provide the eight points denoted as filled in black circles in Fig 4. Of course we still have the comparimetric uncertainty of what should be inserted between the points, but if a power law is suspected, we can assume the smooth monotonic function of the form $f = \beta q^\gamma$ and determine the value of γ from the data.

1.2 Doing the experiment in bulk

With an actual camera, there are multiple pixels, not just one. So rather than exactly doubling the exposure by using a black card to cover half the lamp, suppose that we take two pictures of the same subject matter, the two pictures differing only in exposure. Suppose that one picture is exactly twice the exposure of the other.

Pictures usually consist of a two dimensional lattice of pixels, upon which falls a two dimensional distribution of light $q(x, y)$ for the first picture, and $2q(x, y)$ for the second picture (because the exposure is twice as much for the second picture). Thus the two pictures may be written as:

$$v_f = f(q(x, y)) \quad (3)$$

$$v_g = f(2q(x, y)) = g(q(x, y)). \quad (4)$$

A typical size for a picture is an array that is 480 pixels high and 640 pixels wide (same aspect ratio as standard NTSC television aspect ratio, namely 4:3). Let us consider a simpler example, namely two pictures that are each 3 pixels high and 4 pixels wide:

$$v_f = \begin{bmatrix} 1 & 3 & 2 & 3 \\ 3 & 2 & 1 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}; v_g = \begin{bmatrix} 2 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 \\ 0 & 1 & 3 & 0 \end{bmatrix}. \quad (5)$$

In this example, assume that we have a 2 bit camera, such that it has grey values that go from 0 to 3.

We now introduce the *comparagram*. The comparagram is a two dimensional array of size M by N where M is the number of grey values in the first image, and N is the number of grey values in the second image, where entry $J[m, n]$ is a count of how many times a pixel in image 1 has greyvalue m and the corresponding pixel in image 2 has greyvalue n . In this case both images have 4 grey values, so the comparagram is a 4 by 4 matrix:

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

Summing across rows gives the histogram of the first image:

$$h_f = [3 \quad 2 \quad 4 \quad 3].$$

Summing down columns gives the histogram of the second image,

$$h_g = [2 \quad 1 \quad 5 \quad 4].$$

2 Typical cameras

Some video cameras function much like the photocell, but with a value of $\gamma = 0.54$ instead of the γ values of 0.6 to 0.9 that are typical of photocells.

However, most cameras follow a more complicated law than the simple power law. In particular, various laws such as a simple 2 parameter law $(\frac{q^a}{q^a+1})^c$ have been proposed [3] and used in various research applications such as wearable imaging systems [3], computer vision, and robotics [4]

More generally, we begin without any specific assumptions about the response function, f , by first constructing the comparagram from two differently exposed pictures of the same subject matter as might arise in considering any two successive frames from a video camera or image sequence.

Typically there will be a difference in exposure between two successive frames, due to some amount of camera motion, in the sense that most cameras have some kind of automatic exposure mechanism. Typically, due to noise in images (sensor noise, as well as inter-frame motion artifacts, etc), comparagrams do not provide points that only lie on a slender curve. More typically, comparagrams are defined as a cloud of points clustered along an underlying curve. Thus slenderization of the comparagram [3] is a typical first step in recovery of the comparimetric function, $g(f)$. See Fig. 5

3 Constructing a Comparagram from a Comparagram

Ideally, a comparagram is a function which plots $f(q)$ against $f(2q)$ that appears much like the solid line in Fig. 3. Unfortunately, the comparagram does not usually appear as this ideal comparimetric function. Rather, the comparagram appears as a cloud of points around this comparimetric function. Three methods are commonly used for recovering the underlying comparimetric function from the comparagram:

- first moments may be calculated down each of the columns of the comparagram or across each row of the comparagram. This would correspond to Bayes' Least Error if the comparagram were to be regarded as a joint probability distribution;

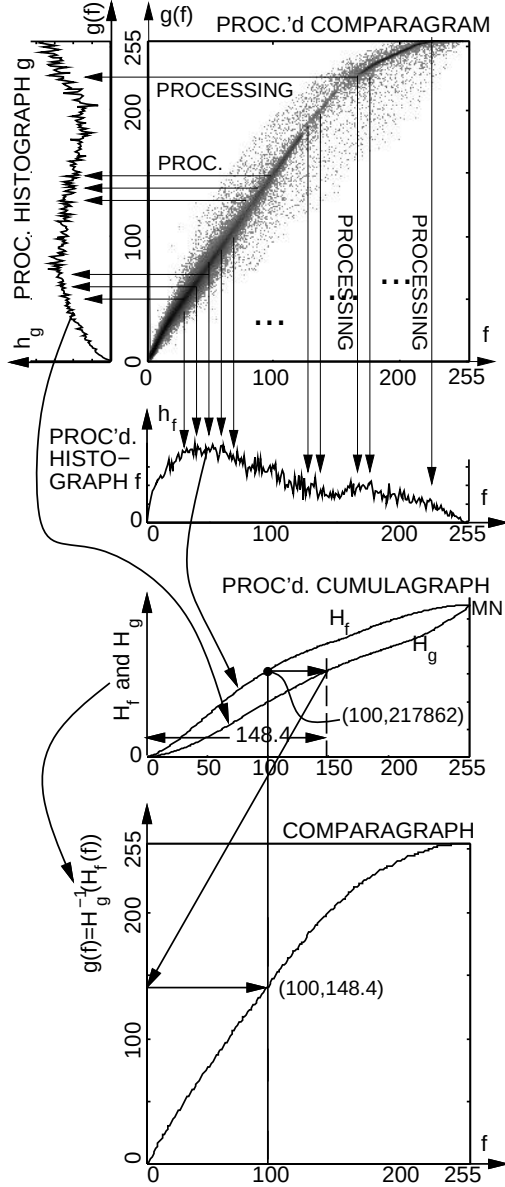


Figure 5: **Slenderizing a comparagram by marginalization:** A comparagram is constructed from the two differently exposed pictures. The dynamic range of comparagrams tends to be quite high, so here the bin counts are displayed on a logarithmic tone scale, $\log(J + \epsilon) - \log(\epsilon)$, $\epsilon > 0$ preventing calculation of $\log(0)$ (and thresholding noise below -60dB or so). Next summing down columns produces the marginal h_f along the f axis. Summing along rows produces the marginal h_g along the g axis. Whatever tone scale is selected for the optimal display of the comparagram (typically logarithmic or power law) will affect h_f and h_g , so h_f and h_g are typically not histograms. We refer to them as *histographs* to make this distinction. Cumulative sums are taken, giving “cumulagraphs”. Notice how both cumulagraphs meet at the right, which is the total number of pixels in either of the two input images. The comparagram is then re-constructed from only its marginals, such that only a slender ridge remains. By also constraining this reconstruction to be monotonic, the result is a graph, and is referred to as a *comparagram* of the underlying *comparametric* function[3]. The arrows connecting the comparagraph and cumulagraphs show an example of one point $g(100)$, e.g. how $H_g^{-1}(H_f(100))$ is calculated. First we find $H_f(100) = 217862$, and then we find the index of H_g having this same value. We notice $H_g(148) = 217785$ and $H_g(149) = 217912$. Interpolating we obtain $g(100) = 148.4$.

- a maximum likelihood estimator can be used to pick the indices of the comparagram which have the highest values as entries. Alternatively a combination of row and column calculations could be taken;
- the comparagram may be determined using only the marginals of the comparagram. This process is called marginalization. The comparagram is slenderized into a comparagram by getting rid of the joint information in the distribution. This is equivalent to estimation of a joint PDF knowing only the marginals. It turns out that throwing away the joint information leads to a good estimate of the comparagram from the comparagram. Prior to marginalizing the comparagram, it should first be processed, by thresholding and tone scale adjustment. This initial processing is important, and dramatically improves the results. Without this initial processing, marginalization is the same as histogram specification. But with this initial processing, the marginals are no longer mere histograms. We refer to the marginals of a processed comparagram as *histographs* to make this distinction. Typically comparagrams are displayed on a thresholded logarithmic scale. Preferably also, any entry in the comparagram that has an entry of 1, is set to 0. We have found this to be the minimum amount of thresholding acceptable.

An set of computer programs has been developed (by Mann, Manders, and Fung) to explore this new kind of image processing in real time. These programs are available for free (freesource under GNU General Public License) from <http://sourceforge.net/projects/comparametric>.

3.1 A simple example of computing the comparagram from the comparagram

In this example, the threshold and tonescale adjustment are omitted for simplicity of understanding the example. It should be noted, however, that results will generally be poor when thresholding is not used, and therefore this example is only meant for illustration.

Looking at the previous example, the marginal of the comparagram for the first image is: $\{3, 2, 4, 3\}$. Since we have ommitted the threshold or tone scale adjustment for teaching purposes, this marginal (histograph) is the same as the histogram of the first image. The corresponding marginal for the second image is $\{2, 1, 5, 4\}$. If each position of the histogram is numbered from 0 to 3, the first histogram has 3 entries in

it's first position. We then ask, "how far must one look into the second histogram to account for all of the data in the first histogram's first position?". To simplify this task the histograms are converted to cumulatives, such that each entry of the cumulative expresses how much of the data is present at each point of the corresponding histogram. This implies that the cumulative of the first histogram is $\{3, 5, 9, 12\}$. The cumulative of the second histogram is $\{2, 3, 8, 12\}$.

Looking at the first entry of the first cumulative, there are 3 data points. We now progress through the second cumulative until all three data points have been accounted for. The first entry in the second cumulative is 2, which does not account for all of the data. The second entry is 3 which accounts for all of the data in the first cumulative entry exactly. For this reason the first cumulative entry is 1 (as 1 is the index of the second cumulative where all of the data has been accounted for). Continuing the process, the second entry in the first cumulative is 5. Looking at the second cumulative, the second entry is 3 and therefore does not account for enough data, however, the third entry is 8, which accounts for too much of the data. Thus, we must interpolate between these two points (1 and 2) to find a reasonable value. Following this method, the third value will fall between 2 and 3, and finally the last value will be exactly 3.

4 A real world example

We test our theory of comparametric imaging, by estimation of the exposures and response function from a sequence of differently exposed pictures of the same (in regions of overlap) subject matter.

Multiple differently exposed images of the same subject matter occur naturally in video sequences, where Automatic Gain Control (AGC) is present, or in cameras having some form of automatic exposure.

This naturally occurring exposure fluctuation allows us to estimate the camera's response function, as well as the exposure differences, as shown in Fig 6. The results of the exposure estimates appear in Table 1.

5 Conclusions

When we ask the fundamental question "what does a camera measure", we arrive at a new concept of quantimetric imaging, with a new quantimetric unit, q , characteristic of a particular camera (e.g. each kind of camera defines its own quantimetric unit q based on its spectral response, etc.).

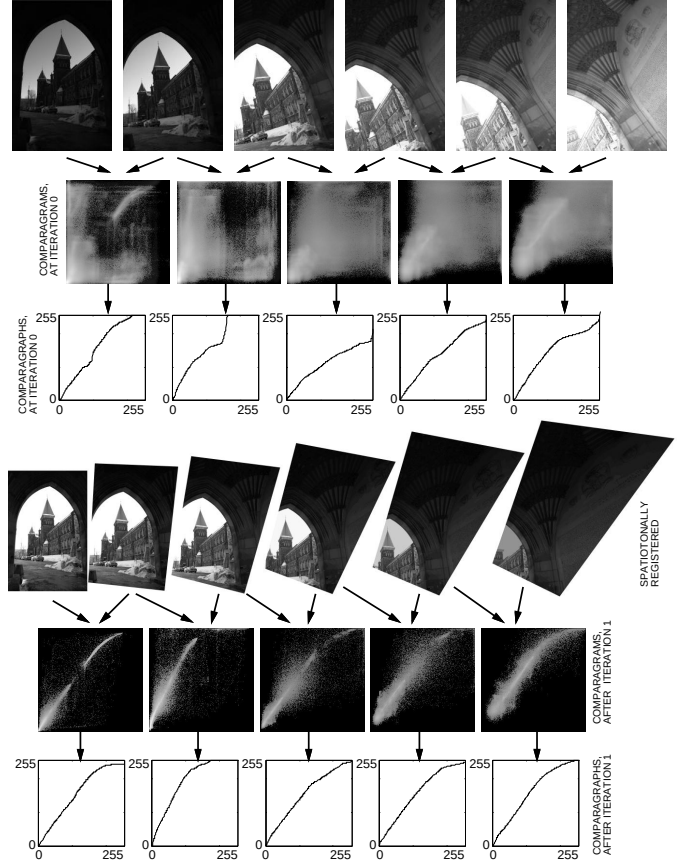


Figure 6: Along the top row we see six differently exposed pictures of the same subject matter arising from changes in camera settings (automatic exposure, etc.). Leftmost: When light colored objects enter the field of view, such as when a bright sky takes up a good portion of the image, the exposure will typically be reduced. Rightmost: When the camera swings toward darker subject matter the reduced light gives rise to an increase in exposure. Thus we have various differently exposed pictures of the same (in regions of overlap) subject matter. Variation in frame-to-frame exposure allows us to estimate the camera's response function, as well as the exposure differences. At zero iterations the pairwise estimates of the comparagrams are quite crude, but after one iteration, they converge to satisfactory plots. Note that the second comparagram reaches higher, because the exposure difference jumped two steps, rather than just one as was the case in the other four comparagrams.

Iteration 0	Iteration 1	True value
2.530293	2.011216	2
3.444645	4.051448	4
2.785258	1.984015	2
2.771012	1.996743	2
2.226825	1.986862	2

Table 1: Results of comparametric interframe exposure estimator applied to the six frames in the image sequence of Fig 6. Although the results are initially crude, after one iteration, the error falls to well under one percent for all five interframe estimates.

Fluctuations in interframe exposures, along a sequence of images, give rise to a *comparametric* relationship between successive pairs of images. This allows us to estimate the response function of the camera (to derive the quantimetric unit q) as well as the relative differences in exposure.

References

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