Problem Set.

7.4 (b) 7.7 7.10 7.26 7.28

Solution

7.4 (b) See Section 7.1.2 for the definition of bilinear transform

$$Z = \frac{1 + T_d/2\, S}{1 - T_d/2\, S} = \frac{1+s}{1-s}$$

$$H_c(s) = \frac{2}{1 - e^{0.2}\frac{1-s}{1+s}} - \frac{1}{1 - e^{-0.4}\frac{1-s}{1+s}}$$

$$= \frac{2}{1+e^{-0.2}} \cdot \frac{s+1}{s + \frac{1-e^{-0.2}}{1+e^{-0.2}}} - \frac{1}{1+e^{-0.4}} \frac{s+1}{s + \frac{1-e^{-0.4}}{1+e^{-0.4}}}$$

7.7 $\omega = \Omega \cdot T$

$\omega_p = 2\pi(1000)10^{-4} = 0.2\pi$ rad.

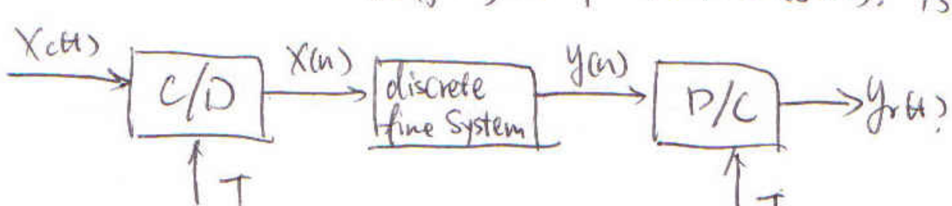
$\omega_s = 2\pi(1100)10^{-4} = 0.22\pi$ rad

Therefore the specification for $H_d(e^{j\omega})$ are.

$0.99 \le |H_d(e^{j\omega})| \le 1.01, \quad 0 \le |\omega| \le 0.20\pi$

$|H_d(e^{j\omega})| \le 0.01, \quad 0.22\pi \le |\omega| \le \pi$

Note the condition $X_c(j\Omega) = 0$ for $|\Omega| \ge 2\pi(5000)$, is the Nyquist Condition.



7.10. Refer to Section 7.1.2 for bilinear transform frequency mapping equation.

$$\omega_c = 2\tan^{-1}\left(\frac{\Omega_c T}{2}\right)$$

$$= 2\tan^{-1}\left(\frac{2\pi(2000)(0.4\times10^{-3})}{2}\right)$$

$$= 0.7589\pi \text{ rad}.$$

7.26.

(a) For impulse invariance.

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\frac{\omega}{T_d} + \frac{2\pi k}{T_d}\right)\right)$$

$$H(e^{j\omega})\Big|_{\omega=0} = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\frac{\omega}{T_d} + \frac{2\pi k}{T_d}\right)\right)\Big|_{\omega=0} = H_c(j\Omega)\Big|_{\Omega=0}$$

$$\Longleftrightarrow \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} H_c\left(j\frac{2\pi k}{T_d}\right) = 0$$

(b) Since the bilinear transform maps $\Omega=0$ to $\omega=0$, the condition will hold for any choice of $H_c(j\Omega)$

7.28 (a) $s = \dfrac{1-z^{-1}}{1+z^{-1}} \Rightarrow j\Omega = \dfrac{1-e^{-j\omega}}{1+e^{-j\omega}} \Rightarrow \Omega = \tan\left(\dfrac{\omega}{2}\right)$

$$\Rightarrow \omega_{p_1} = 2\tan^{-1}\Omega_p$$

(b). $s = \dfrac{1+z^{-1}}{1-z^{-1}} \Rightarrow j\Omega = \dfrac{1+e^{-j\omega}}{1-e^{-j\omega}} \Rightarrow \Omega = \tan\left(\dfrac{\omega-\pi}{2}\right)$

$$\Rightarrow \omega_{p_2} = \pi + 2\tan^{-1}\Omega_p$$

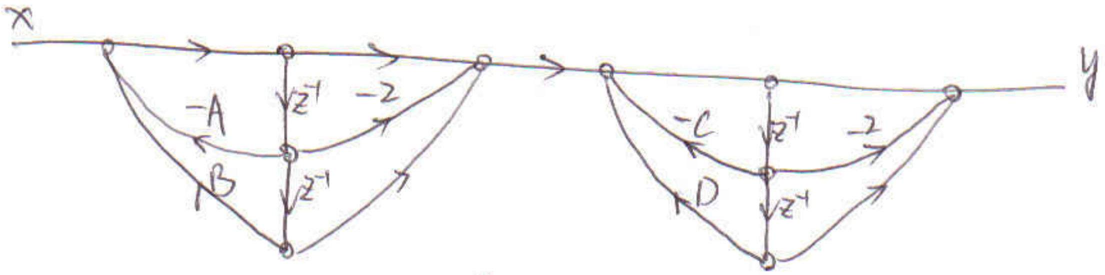
(c) $\tan\left(\frac{\omega_{p_2}-\pi}{2}\right) = \tan\left(\frac{\omega_{p_1}}{2}\right)$

$\Rightarrow \omega_{p_2} = \omega_{p_1} + \pi$

(d) $H_2(z) = H_1(-z)$

$A \to -A \quad B \to B \quad C \to -C \quad D \to D$

$2 \to -2$



$H_2(z)$