

Problem Set

5.15 5.23 5.25 6.26 6.29

5.15 (a) $H(e^{j\omega}) = 2 + e^{-j\omega} + 2e^{-2j\omega}$
 $= (1 + 4\cos\omega)e^{-j\omega}$
Generalized Linear Phase filter.

(b) This sequence has no even or odd symmetry
So, it is neither generalized linear or linear.

(c) $H(e^{j\omega}) = 1 + 3e^{-j\omega} + e^{-2j\omega}$
 $= (3 + 2\cos\omega)e^{-j\omega}$

it is linear Phase.

(d) $H(e^{j\omega}) = 1 + e^{-j\omega}$
 $= 2\cos(\omega/2)e^{-j\omega/2}$

it is generalised linear phase

(e) $H(e^{j\omega}) = 1 - e^{-2j\omega}$
 $= (2\sin\omega)e^{-j\omega + j\frac{\pi}{2}}$

it is generalised linear Phase

S.23

(a) Type I:
$$A(\omega) = \sum_{n=0}^{M/2} a[n] \cos \omega n$$

There is no restriction on $\omega = 0$ or π

~~II~~ Type II:
$$A(\omega) = \sum_{n=1}^{(M+1)/2} b[n] \cos \omega(n - \frac{1}{2})$$

$\cos \omega(n - \frac{1}{2}) \Big|_{\omega=\pi} = \cos(n\pi - \frac{\pi}{2}) = 0, \Rightarrow H(e^{j\pi}) = 0$

There is no restriction on $\omega = 0$

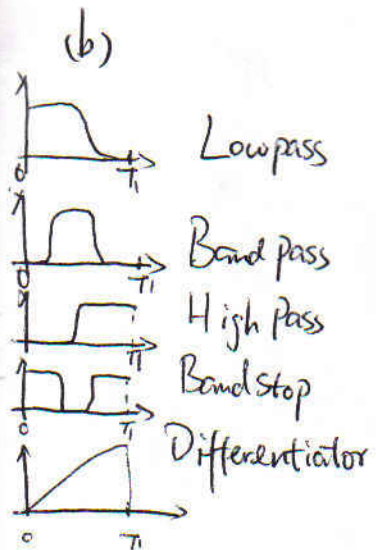
~~III~~ Type III:
$$A(\omega) = \sum_{n=0}^{M/2} c[n] \sin \omega n$$

$\sin \omega n \Big|_{\omega=0} = \sin \omega n \Big|_{\omega=\pi} = 0 \Rightarrow H(e^{j0}) = H(e^{j\pi}) = 0$

Type IV:
$$A(\omega) = \sum_{n=1}^{(M+1)/2} d[n] \sin \omega(n - \frac{1}{2})$$

$\sin \omega n \Big|_{\omega=0} = \sin \omega(n - \frac{1}{2}) \Big|_{\omega=0} = 0 \Rightarrow H(e^{j0}) = 0$

There is no restriction on $\omega = \pi$



	TYPE I	TYPE II	TYPE III	TYPE IV
Lowpass	Y	Y	Y N	N
Band pass	Y	Y	Y	Y
High Pass	Y	N	N	Y
Band stop	Y	N	N	N
Differentiator	Y	N	N	Y

25. The statement is false. The noncausal system can have a positive constant group delay.

Consider the causal FIR system

$$h[n] = \delta[n] + \delta[n-1] + 2\delta[n-2] + \delta[n-3] + \delta[n-4]$$

it is a linear phased system, with the positive constant group delay as '2'.

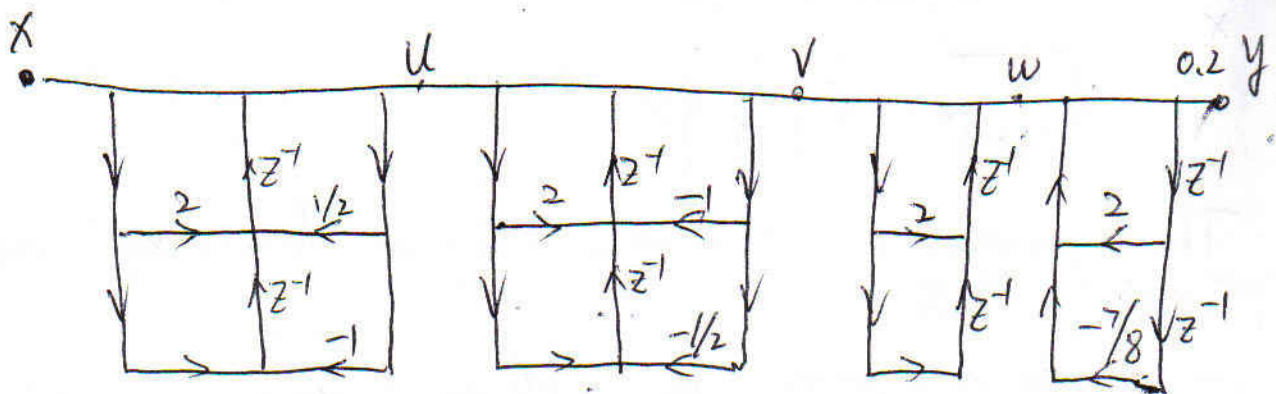
Now, consider

$$\begin{aligned} h_1[n] &= \delta[n+1] + \delta[n] + 2\delta[n-1] + \delta[n-2] + \delta[n-3] \\ &= h[n+1] \end{aligned}$$

it is a noncausal system. But since $h_1[n] = h[n+1]$, $h_1[n]$ is also a linear phase system, with the positive constant group delay as '1'.

6.26.

$$(a) \quad H(z) = \frac{(1+z^{-1})^2}{1-\frac{1}{2}z^{-1}+z^{-2}} \cdot \frac{(1+z^{-1})^2}{1+z^{-1}+\frac{1}{2}z^{-2}} \cdot \frac{1}{(1+z^{-1})^2} \cdot \frac{1}{1-2z^{-1}+\frac{7}{8}z^{-2}} \cdot a^2$$



It is not unique, since the order of 2-order denominator can be rearranged.

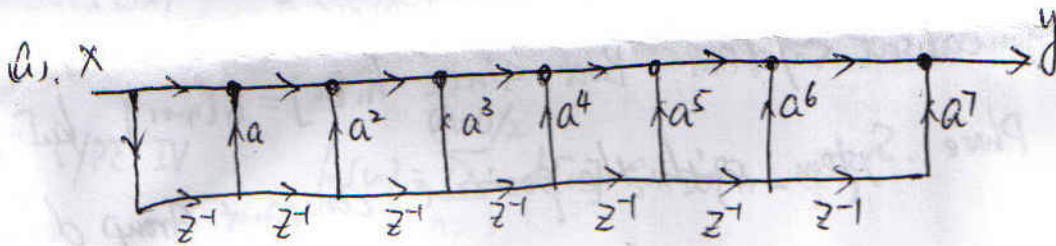
b)
$$u[n] = x[n] + 2x[n-1] + x[n-2] + \frac{1}{2}u[n-1] - u[n-2]$$

$$V[n] = u[n] + 2u[n-1] + u[n-2] - V[n-1] - \frac{1}{2}V[n-2]$$

$$W[n] = V[n] + 2V[n-1] + V[n-2]$$

$$Y[n] = 0.2W[n] + 2Y[n-1] - \frac{7}{8}Y[n-2]$$

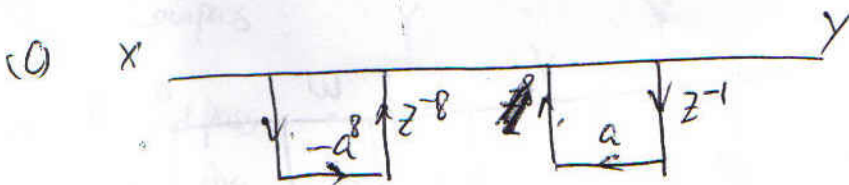
6.29



b) Geometrical Series

$$\sum_{n=0}^7 a^n z^{-n} = \frac{1 - a^8 z^{-8}}{1 - a z^{-1}}$$

The ROC is $|z| > |a|$, since it is a causal system



d) The implementation in part (c) is recursive, however the overall system is FIR

e) (c) has the most storage, (a) has the most arithmetic.