ECE 431 Problem Set. Solution

6.3 6.5 6.6 6.10 ✓ 6.23 ✓ 6.25 ✓ 6.26

6.3

$$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1+\frac{3}{4}z^{-1}}$$

$$= \frac{2+\frac{1}{4}z^{-1}}{1+\frac{1}{4}z^{-1}-\frac{3}{8}z^{-2}}$$

(d) is the transposed direct from II implementation of H(z)

6.5.

(a) & (b)

$$\frac{Y(z)}{X(z)} = \frac{1}{1-z^{-1}-2z^{-2}} \cdot \frac{1}{1-3z^{-1}-z^{-2}}$$

$$= \frac{1}{1-4z^{-1}+7z^{-3}+2z^{-4}}$$

$$x(n) = y(n) - 4y(n-1) + 7y(n-3) + 2y(n-4)$$

(c) 4 real adds and 2 real multiplies in the implementation.

(d) it is not possible. due to the order of the H(z) denominator is 4

6.6.

(a) $$H(z) = 1 - 2z^{-1} + 4z^{-2} + 3z^{-3} - 1z^{-4} + z^{-5}$$

$$h(n) = \delta(n) - 2\delta(n-1) + 4\delta(n-2) + 3\delta(n-3) - \delta(n-4) + \delta(n-5)$$

(b). $$H(z) = 1 - z^{-1} + 3z^{-2} + 4z^{-3} - 2z^{-4} + z^{-5}$$

$$h(n) = \delta(n) - \cancel{4}\delta(n-1) + 3\delta(n-2) + 4\delta(n-3) - 2\delta(n-4) + \delta(n-5)$$

(c)

$$H(z) = 2\cdot(1 + z^{-7}) + 3(z^{-1} + z^{-6}) - (z^{-2} + z^{-5}) + (z^{-3} + z^{-4})$$

$$h(n) = 2\delta(n) + 3\delta(n-1) - \delta(n-2) + \delta(n-3) + \delta(n-4) - \delta(n-5) + 3\delta(n-6) + 2\delta(n-7)$$

(d). $$H(z) = (1 + z^{-6}) + 2(z^{-1} + z^{-5}) - (z^{-2} + z^{-4}) + 3z^{-3}$$

$$h(n) = \delta(n) + 2\delta(n-1) - \delta(n-2) + 3\delta(n-3) - \delta(n-4) + 2\delta(n-5) + \delta(n-6)$$

6.10. (a) $$w(n) = \tfrac{1}{2}y(n) + x(n)$$

$$v(n) = \tfrac{1}{2}y(n) + 2x(n) + w(n-1)$$

$$y(n) = v(n-1) + x(n)$$

(b) $$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}} = \frac{(1+z^{-1})(1+z^{-1})}{(1+\frac{1}{2}z^{-1})(1-z^{-1})}$$

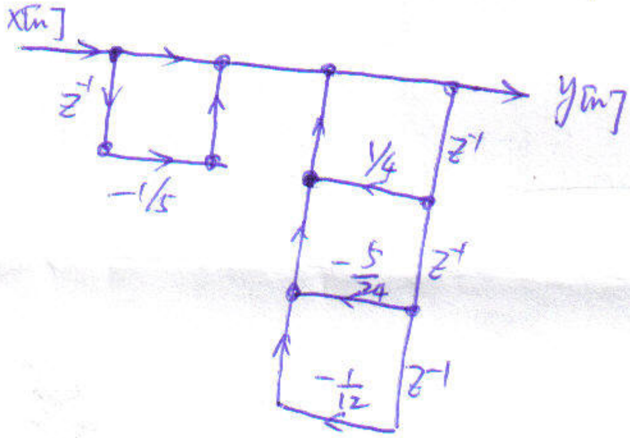
(c) Not stable.

Since the two poles are $-\frac{1}{2}$ and 1, ~~which is inside the unit circle~~. '1' is on the unit circle.

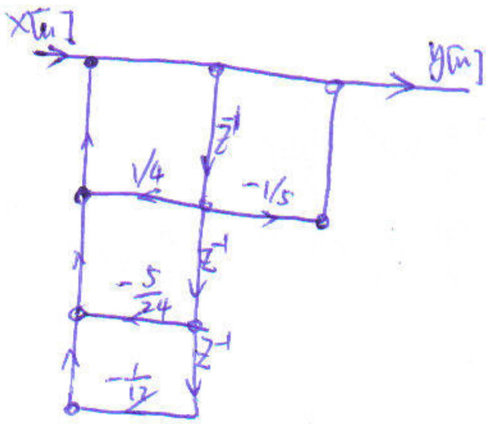
6.23 (a)

<i>

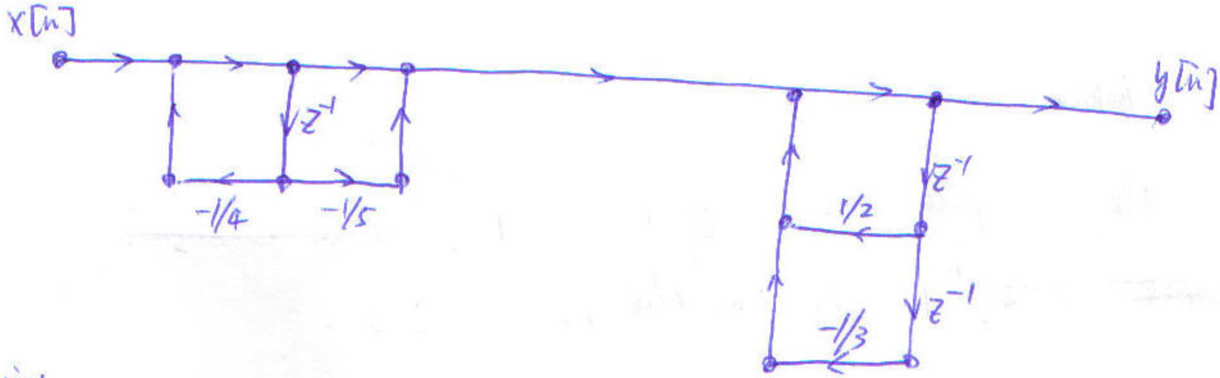
$$H(z) = \frac{1 - \frac{1}{5}z^{-1}}{1 - \frac{1}{4}z^{-1} + \frac{5}{24}z^{-2} + \frac{1}{12}z^{-3}}$$



<ii>

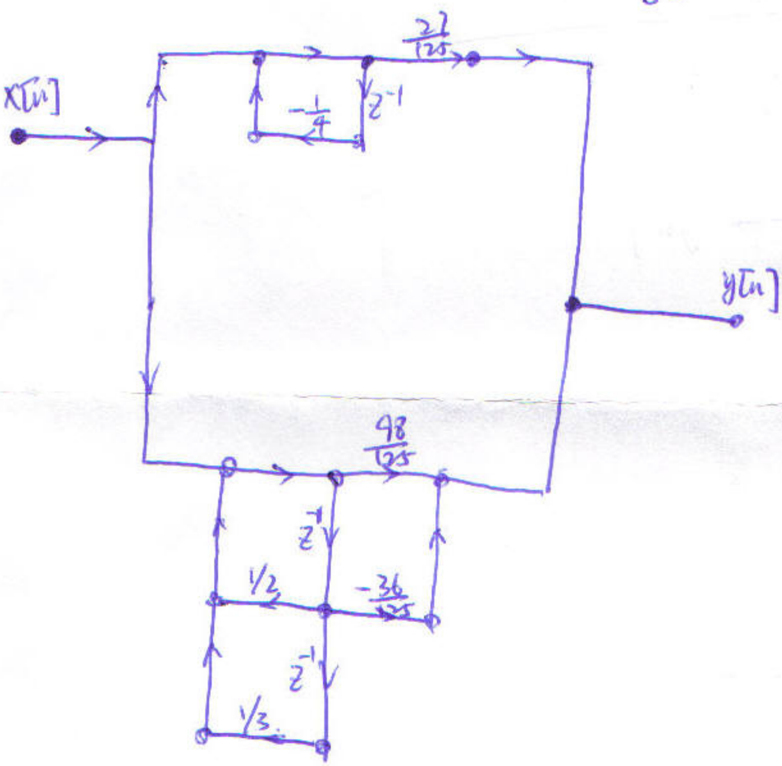


<iii> $H(z) = \frac{1 - \frac{1}{5}z^{-1}}{1 + \frac{1}{4}z^{-1}} \cdot \frac{1}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}}$



<iv>

$$H(z) = \frac{\frac{27}{125}}{1+\frac{1}{4}z^{-1}} + \frac{\frac{98}{125} - \frac{36}{125}z^{-1}}{1-\frac{1}{2}z^{-1}-\frac{1}{3}z^{-2}}$$



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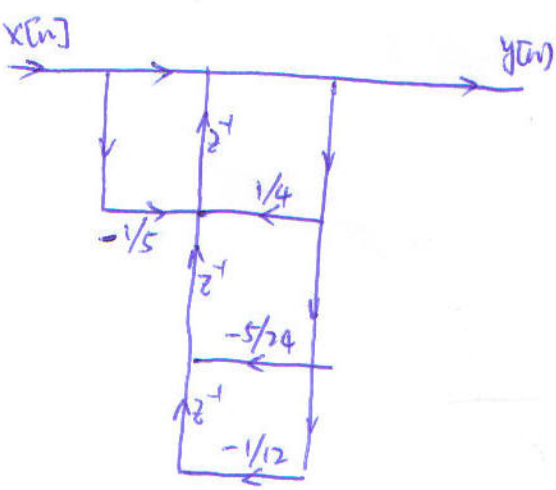


(b). $y[n] - \frac{1}{4} y[n-1] + \frac{5}{24} y[n-2] + \frac{1}{12} y[n-3] = x[n] - \frac{1}{5} x[n-1]$

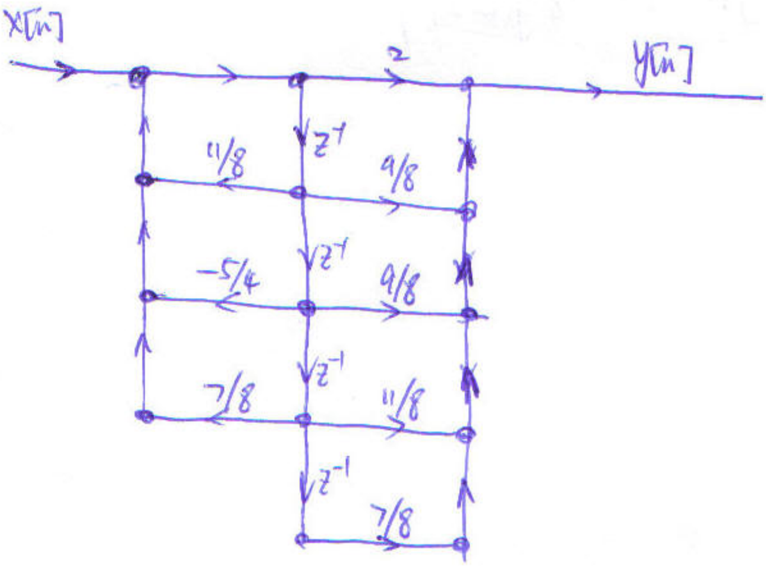
6.25.

(a) $H(z) = \frac{1}{1 - z^{-1}} \cdot \left[\frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{3}{8}z^{-1} + \frac{7}{8}z^{-2}} + 1 + 2z^{-1} + z^{-2} \right]$

$$= \frac{2 + \frac{9}{8}z^{-1} + \frac{9}{8}z^{-2} + \frac{11}{8}z^{-3} + \frac{7}{8}z^{-4}}{1 - \frac{11}{8}z^{-1} + \frac{5}{4}z^{-2} - \frac{7}{8}z^{-3}}$$

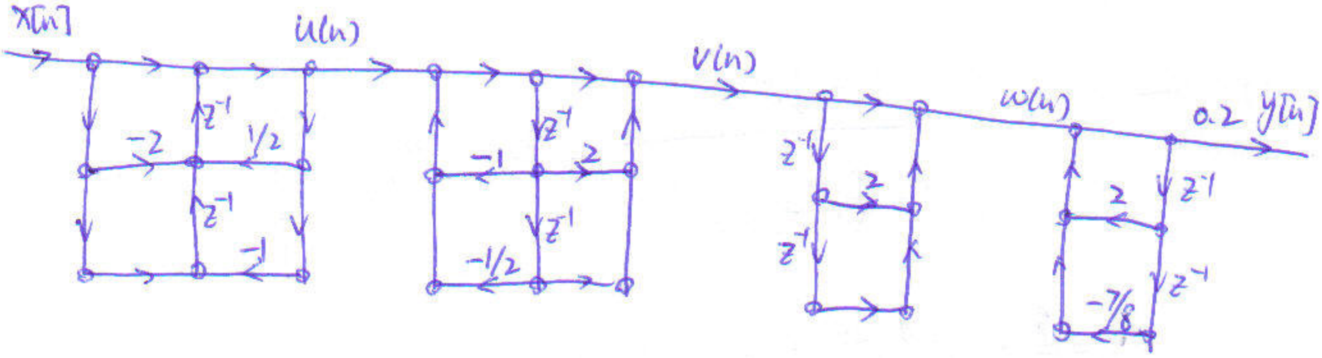
(b) $y[n] - \frac{11}{8} y[n-1] + \frac{5}{4} y[n-2] - \frac{7}{8} y[n-3] = 2x[n] + \frac{9}{8} x[n-1] + \frac{9}{8} x[n-2] + \frac{11}{8} x[n-3] + \frac{7}{8} x[n-4]$

(c).



6.26

a) $$H(z) = \frac{(1-z^{-1})^2}{1-\frac{1}{2}z^{-1}+z^{-2}} \cdot \frac{(1+z^{-1})^2}{1+z^{-1}+\frac{1}{2}z^{-2}} \cdot (1+z^{-1})^2 \cdot \frac{1}{1-2z^{-1}+\frac{7}{8}z^{-2}} \cdot 0.2$$



b)

$$u(n) - \frac{1}{2}u(n-1) + u(n-2) = x(n) + x(n-2) + 2x(n-1)$$

$$v(n) = u(n) - v(n-1) - \frac{1}{2}v(n-2) + 2u(n-1) + u(n-2)$$

$$w(n) = v(n) + 2v(n-1) + v(n-2)$$

$$y[n] = 0.2 \cdot (w(n) + 2y[n-1] - 7/8\, y[n-2])$$