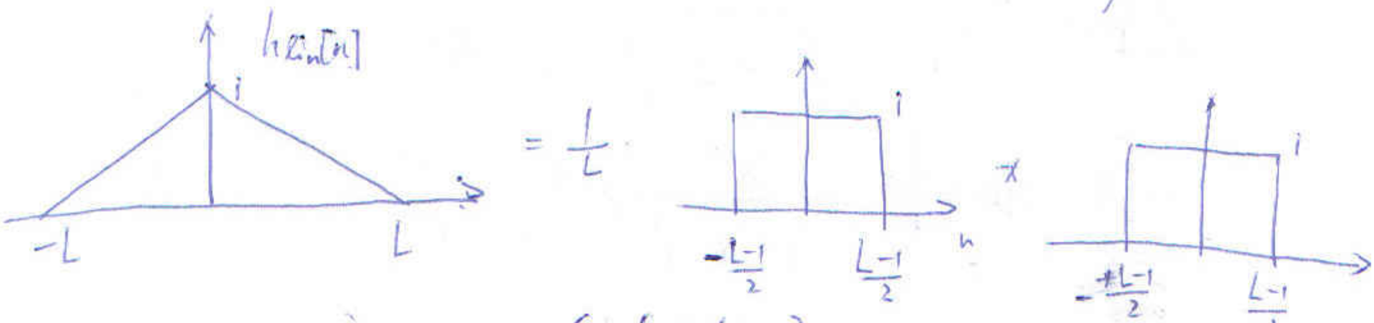
ECE431F Problem Set 5.

4.50(b) 4.51 ✓ 3.2 3.9 3.21 3.23 ✓ 3.47 ✓

Solution:

4.50(b) $h_{lin}[n]$ corresponds to the convolution of two rectangular.



$$H_{lin}[e^{j\omega}] = \frac{1}{L}\left(\frac{\sin(\omega L/2)}{\sin(\omega/2)}\right)^2$$

4.51. $\phi_{xx}(e^{j\omega}) = \text{DTFT}[x[n] * x[-n]]$

$$= X(e^{j\omega}) \cdot X^*(e^{j\omega})$$

$$= |X(e^{j\omega})|^2$$

So the bandwidth of $\phi_{xx}(e^{j\omega})$ is the same as $X(e^{j\omega})$

$\phi_1[n] = \phi_3[n]$, if and only if $H_2[e^{j\omega}]$ is an ideal lowpass filter with a cutoff of π/L

3.2 $x[n] = n\,u[n] - (n-N)\,u[n-N]$

where $u[n] = \begin{cases} 0 & n<0 \\ 1 & n\geq 0 \end{cases}$

$$Z[n\,u[n]] = \frac{z^{-1}}{(1-z^{-1})^2}, \quad |z|>1$$

$$Z[(n-N)u[n-N]] = \frac{z^{-1}}{(1-z^{-1})^2} z^{-N}, \quad |z|>1$$

therefore, $X(z) = \dfrac{z^{-1}(1-z^{-N})}{(1-z^{-1})^2}$

3.9 (a). $H(z) = \dfrac{2}{1-\frac{1}{2}z^{-1}} - \dfrac{1}{1+\frac{1}{4}z^{-1}}$

ROC $\Rightarrow |z| > \frac{1}{2}$, Since $H(z)$ is Causal.

(b). Stable, because the unit circle is included in the ROC.

(c) $y[n] = -\frac{1}{3}\left(-\frac{1}{4}\right)^n u[n] - \frac{4}{3}(2)^n u[-n-1]$

$$Y(z) = \frac{-\frac{1}{3}}{1+\frac{1}{4}z^{-1}} + \frac{4}{3}\frac{1}{1-2z^{-1}} \qquad \frac{1}{4} < |z| < 2.$$

$$= \frac{1+z^{-1}}{(1+\frac{1}{4}z^{-1})(1-2z^{-1})}$$

$$X(z) = \frac{Y(z)}{H(z)} = \frac{1-\frac{1}{2}z^{-1}}{1-2z^{-1}} \quad |z|<2$$

$$= \frac{1}{1-2z^{-1}} - \frac{1}{2}\frac{z^{-1}}{1-2z^{-1}}$$

$$x[n] = -(2)^n u[-n-1] + \frac{1}{2}(2)^{n-1} u[n]$$

(d) $h[n] = 2\left(\frac{1}{2}\right)^n u[n] - \left(-\frac{1}{4}\right)^n u[n]$

3.21. (a) $y[n]=0 \quad n<0$

$$y[n] = \sum_{k=0}^{n} x[k]\,h[n-k] = \sum_{k=0}^{n} a^{n-k} = \frac{1-a^{n+1}}{1-a} \quad 0\le n < N-1$$

$$y[n] = \sum_{k=0}^{N-1} x[k]\,h[n-k] = \sum_{k=0}^{N-1} a^{n-k} = a^{n+1}\,\frac{1-a^{-N}}{a-1} \quad n\ge N$$

(b)
$$H(z) = \frac{1}{1-az^{-1}} \quad |z|>|a|.$$

$$X(z) = \frac{1-z^{-N}}{1-z^{-1}} \quad |z|>0$$

Therefore. $Y(z) = \dfrac{1-z^{-N}}{(1-az^{-1})(1-z^{-1})} \quad |z|>|a|.$

$$= \left(\frac{1}{1-a}\right)\left(\frac{1}{1-z^{-1}} - \frac{a}{1-az^{-1}}\right)(1-z^{-N})$$

$$y[n] = \left(\frac{1}{1-a}\right)\left[u[n] - a^{n+1}u[n] - u[n-N] + a^{n-N+1}u[n-N] \right]$$

$$= \begin{cases} 0 & n < 0 \\ \dfrac{1-a^{n+1}}{1-a} & 0 \le n \le N-1 \\ a^{n+1} \cdot \dfrac{1-a^{-N}}{a-1} & n \ge N \end{cases}$$

3.23.

(a) $H(z) = -4 - \dfrac{2}{1-\frac{1}{2}z^{-1}} + \dfrac{7}{1-\frac{1}{4}z^{-1}}$

$$h[n] = -4\delta[n] - 2\left(\tfrac{1}{2}\right)^n u[n] + 7\left(\tfrac{1}{4}\right)^n u[n]$$

(b). $y[n] - \frac{3}{4}[n-1] + \frac{1}{8}y[n-2] = x[n] - \frac{1}{2}x[n-2]$

$$\Updownarrow$$

$$Y(z)\cdot\left(1 - \tfrac{3}{4}z^{-1} + \tfrac{1}{8}z^{-2}\right) = X(z)\cdot\left(1 - \tfrac{1}{2}z^{-2}\right)$$

3.47

(a) $X(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n}$

$$X(\infty) = \lim_{z\to\infty} \sum_{n=0}^{\infty} x[n] z^{-n} = x[0]$$

$X(\infty)$ is neither a zero or a pole by the assumption.

(b). $X(z) = k \cdot z^L \dfrac{\prod_{k=1}^{M}(z-c_k)}{\prod_{k=1}^{N}(z-d_k)}$

$L > 0 \Rightarrow$ there is L zeros at $z = 0$

$L < 0 \Rightarrow$ there is L poles at $z = 0$.

Since $X(\infty)$ is neither a pole or zero.

there is $L + M = N$, for $L > 0$

$\cancel{L = N + M}$
$M = -L + N$, for $L < 0$.

in both conditions. the number of poles equals the number of zeros