# ECE431 Problem Set 4 Solution

9.1 9.5 9.22 9.31 9.32

9.1 Define $f(n) = \sum_{k=0}^{N-1} X^*[k] e^{-j2\pi kn/N}$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi kn}{N}}$$

$$= \frac{1}{N} f^*(n)$$

9.5

$$(A-B)D + (C-D)A = AD - BD + AC - AD = AC - BD = X$$

$$(A-B)D + (C+D)B = AD - BD + BC + BD = AD + BC = Y$$

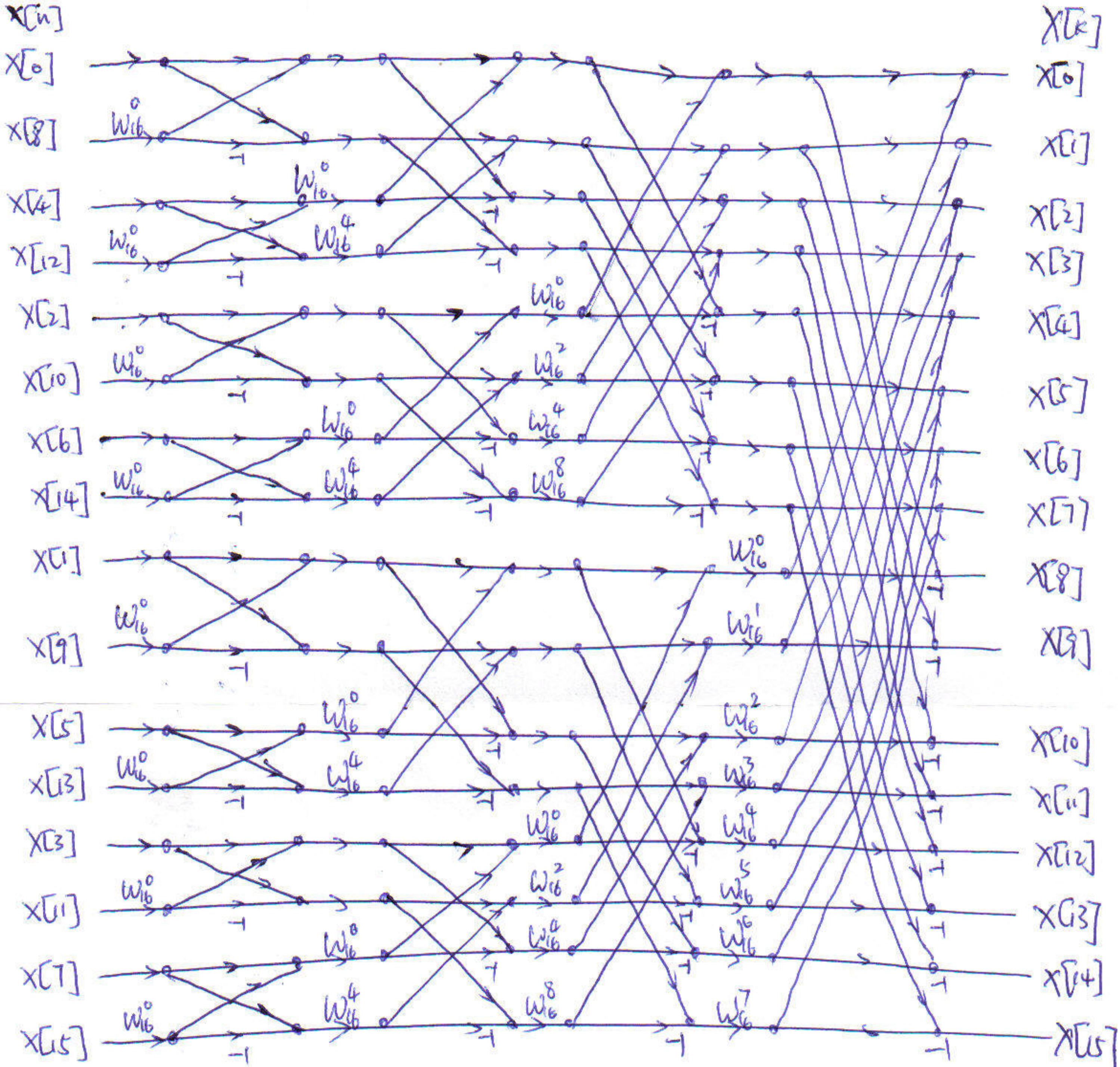
9.22. Page 2.

9.31 (a) if $x[n] = x^*[n] \Rightarrow X[k] = X^*[N-k]$

That is $X_R[k] = X_R[N-k]$ and $X_I[k] = -X_I[N-k]$

$X_R[k] = X_{ER}[k]$ $\quad X_I[k] = X_{OI}[k]$

9.22



x[n]
x[0]
x[8]
x[4]
x[12]
x[2]
x[10]
x[6]
x[14]
x[1]
x[9]
x[5]
x[13]
x[3]
x[11]
x[7]
x[15]
X[k]
X[0]
X[1]
X[2]
X[3]
X[4]
X[5]
X[6]
X[7]
X[8]
X[9]
X[10]
X[11]
X[12]
X[13]
X[14]
X[15]
$W_{16}^0$
$W_{16}^1$
$W_{16}^2$
$W_{16}^3$
$W_{16}^4$
$W_{16}^5$
$W_{16}^6$
$W_{16}^7$
$W_{16}^8$
-1

9.31 (b) $G[k] = X_1[k] + jX_2[k]$

due to the result of (a) $= (X_{1ER}[k] + jX_{1OI}[k]) + j(X_{2ER}[k] + jX_{2OI}[k])$

$= X_{1ER}[k] - X_{2OI}[k] + j(X_{1OI}[k] + X_{2ER}[k])$

So. $G_{ER}[k] = X_{1ER}[k]$ ; $G_{OR}[k] = -X_{2OI}[k]$;

$G_{EI}[k] = X_{2ER}[k]$ ; $G_{OI}[k] = X_{1OI}[k]$;

$X_1[k] = G_{ER}[k] + jG_{OI}[k] = X_{1ER}[k] + jX_{1OI}[k]$

$X_2[k] = G_{EI}[k] - jG_{OR}[k] = X_{2ER}[k] + jX_{2OI}[k]$

(c). $N = 2^v$ point FFT requires $N/2 \log_2 N$ complex multiplications and $N \log_2 N$ complex additions. This is $2N \log_2 N$ real multis and $3N \log_2 N$ real additions

<i> 2 times ~~N-DFT~~ N-FFT is $4N \log_2 N$ real multis and. $6N \log_2 N$ real additions

<ii>

| | #real multi | #real add. |
|---|---|---|
| N point FFT getting G[k] | $2N \log_2 N$ | $3N \log_2 N$ |
| Computing $G_{ER}$ $G_{EI}$ $G_{OI}$ $G_{OR}$ | ~~4N~~ 2N | ~~4N~~ 2N |
| Computing $X_1[k]$ $X_2[k]$ ~~from~~ | 0 | 0 + |
| | $2N \log_2 N + 2N$ | $3N \log_2 N + 2N$ |

d).

$$X[k] = \sum_{n=0}^{N-1} x[n] \otimes e^{-j2\pi kn/N}$$

$$= \sum_{n \text{ even}} x[n] e^{-j2\pi kn/N} + \sum_{n \text{ odd}} x[n] e^{-j2\pi kn/N}$$

$$= \sum_{l=0}^{N/2-1} x[2l] e^{-j2\pi kl/(N/2)} + e^{-j2\pi k/N} \cdot \sum_{l=0}^{N/2-1} x[2l+1] e^{-j2\pi kl/(N/2)}$$

$$= \begin{cases} X_1[k] + e^{-j2\pi k/N} \cdot X_2[k] & 0 \le k < \frac{N}{2} \\ X_1[k-N/2] + e^{-j2\pi k/N} X_2[k-\frac{N}{2}] & \frac{N}{2} \le k < N \end{cases}$$

e).

Step 1. $g[n] = x[2n] + j\, x[2n+1]$ of the length $N/2$.

Step 2. $N/2$ FFT $g[n] \longrightarrow G[k]$

Step 3. obtain $G_{OR}[k]$ $G_{ER}[k]$ $G_{OI}[k]$ $G_{EI}[k]$ from $G[k]$

Step 4. Obtain $X_1[k] = DFT[\, x[2n]\, ] = G_{ER}[k] + j\, G_{OI}[k]$

$X_2[k] = DFT[\, x[2n+1]\, ] = G_{EI}[k] - j\, G_{OR}[k]$

from the result of (b)

Step 5 $X[k] = X_1[k] + e^{-j2\pi k/N} \cdot X_2[k]$ by the result of (d)

$\otimes$ $0 \le k < \frac{N}{2}$

$X[k] = X^*[N-k]$ $\frac{N}{2} \le k < N$

| | real multi | real add |
|---|---|---|
| Step 1 | $0$ | $0$ |
| Step 2. | $N \log_2 \frac{N}{2}$ | $\frac{3}{2} N \log_2 \frac{N}{2}$ |
| Step 3 | $N$ | $N$ |
| Step 4 | $0$ | $0$ |
| Step 5. | $2N$ | $2N$ |

$+$

| | |
|---|---|
| $3N + N\log_2 N - N$ | $3N + \frac{3}{2} N \log_2 N - 1.5N$ |
| $= 2N + N\log_2 N$ | $= 1.5N + 1.5N\log_2 N$ |

9.32 (a) $L+P-1 \longrightarrow$ legth of $y[n]$

(b) Suppose $P < L$

The first $P$ points of $y[n]$ need. $n$ times multi $n$ times add

for. $P < n \leq L$ it is $P$ times muti $P$ times add

for $L < n \leq L+P-1$ it is $L+P-n$ times muti $L+P-n$ times add

So for $P < L$

$$\# \text{ multi} = \sum_{k=1}^{P} k + (L-P)P + \sum_{k=L+1}^{L+P-1} L+P-k$$

$$= \frac{P(P+1)}{2} + (L-P)P + \frac{P(P-1)}{2}$$

$$= L \cdot P.$$

$\# \text{ add} = \# \text{ multi}$

if $P > N$ due to the symmetry, same result hold.

(c). Step 1 Compute $N$-DFT of $x[n]$ and $h[n]$

Step 2 $Y[k] = H[k] \cdot X[k]$

Step 3 $y[n] = \text{IDFT}\left[Y[k]\right]$

Note $N \geq L+P-1$ to avoid alaising

(d)

| | # real multi |
|---|---|
| DFT of $x[n]$ | $2N \log_2 N$ |
| DFT of $h[n]$ | $2N \log_2 N$ |
| $Y[k] = X[k]H[k]$ | $4N$ |
| + IDFT | $2N \log_2 N$ |
| | $6N \log_2 N + 4N$ |

| N | Direct | FFT |
|---|---|---|
| 2 | 1 | 20 |
| 4 | 4 | 64 |
| 8 | 16 | 176 |
| 16 | 64 | 448 |
| 32 | 256 | 1088 |
| 64 | 1024 | 2560 |
| 128 | 4096 | 5888 |
| 256 | 16384 | 13312 |