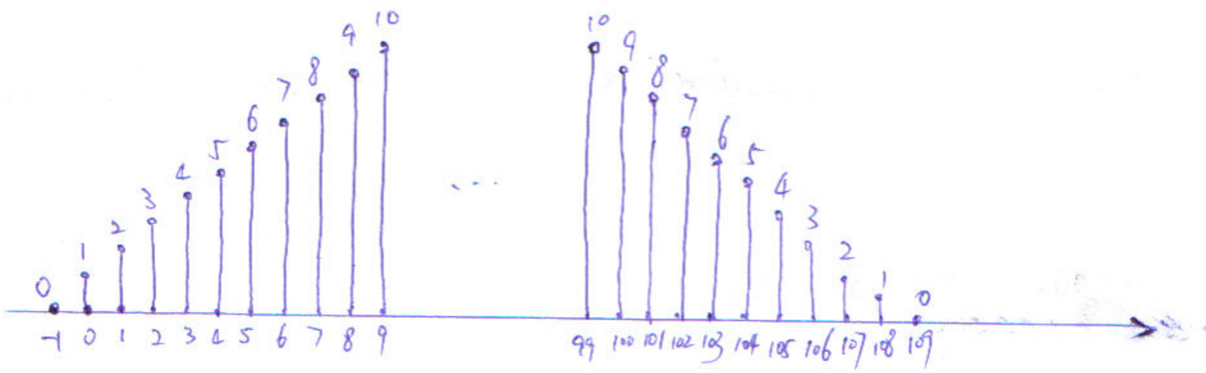
ECE 431F Problem Set 3

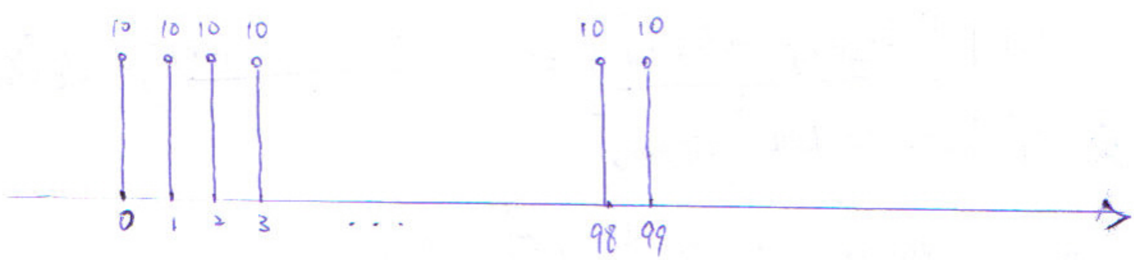
8.27 8.29 8.34 8.35 8.41 8.43 8.48

Solution

8.27 (a) Length $100+10-1 = 109$

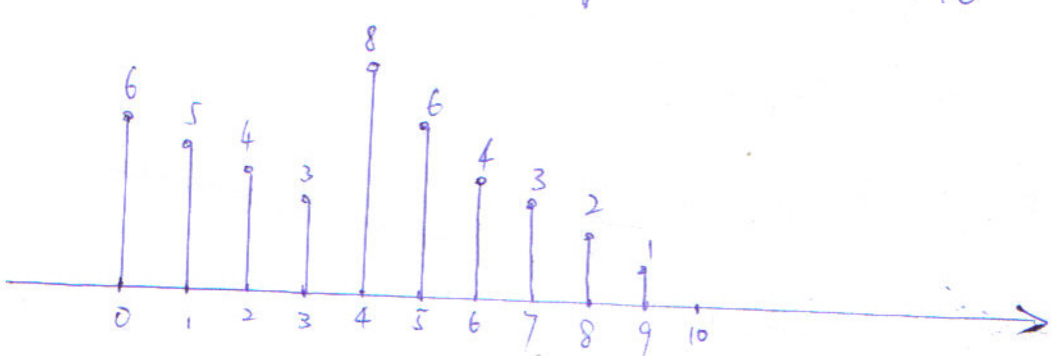


(b) length 100 circular convolution

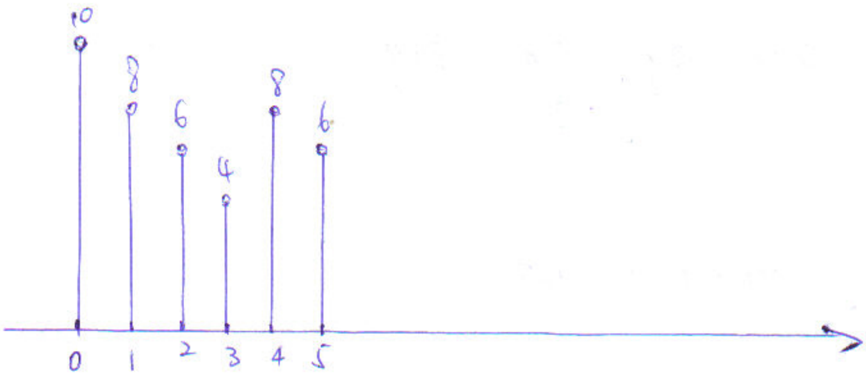


(c) Same as linear convolution

8.29. Linear convolution Length $= 6+5-1 = 10$



(a) $N=6$ Circular convolution



(b) $N=10$ Circular convolution is identical to linear convolution.

8.34 Note that for FT

$$\mathcal{F}\left[\frac{x(t)+x(-t)}{2}\right] = \left[X(j\Omega)+X^*(j\Omega)\right]/2 = \mathrm{Re}(X(j\Omega))$$

Correspondly for DFT

$$\mathrm{DFT}\left[\frac{\overset{x((n))_N}{\cancel{(x(n))_N}} + \overset{x((-n))_N}{\cancel{(x(-n))_N}}}{2}\right] = \mathrm{DFT}\left(\frac{x(n)+x(N-n)}{2}\right) = \mathrm{Re}(X[k])$$

This gives the answer to both questions

(a) No. Because the periodicity of DFT

$$X_e[k] = \sum_{-N+1}^{N+1}\left(\frac{x[n]+x[-n]}{2}\right)W_{2N-1}^{kn} \qquad (-N+1) \le k \le (N-1)$$

$$= \sum_{n=0}^{N-1}\frac{x[n]}{2}W_{2N-1}^{-kn} + \sum_{n=0}^{N-1}\frac{x[n]}{2}W_{2N-1}^{kn}$$

$$= \sum_{n=0}^{N-1}x[n]\cos\left(\frac{2\pi kn}{2N-1}\right)$$

$$\mathrm{Re}[X[k]] = \sum_{n=0}^{N-1}x[n]\cos\left(\frac{2\pi kn}{N}\right)$$

(b) $\frac{1}{2}x[n] + \frac{1}{2}x[N-n]$

8.35

From condition 1.

$$X(e^{j\omega}) = 1 + \frac{A_1}{2}(e^{-j\omega} + e^{j\omega}) + \frac{A_2}{2}(e^{j2\omega} + e^{-j2\omega})$$

$$x[n] = \delta(n) + \frac{A_1}{2}[\delta[n-1] + \delta(n+1)] + \frac{A_2}{2}[\delta(n-2) + \delta(n+2)]$$

From condition 2.

$$x[n-3]\Big|_{n=2} = 5 \Rightarrow x[-1] = 5 \Rightarrow A_1 = 10$$

$$\cancel{\sum_{m=0}^{7} \tilde{w}[m]\tilde{x}[n-3-m]}$$

From condition 3

$$\sum_{m=0}^{7} w[m]\, x[((n-3-m))_8]\Big|_{n=2} = 11$$

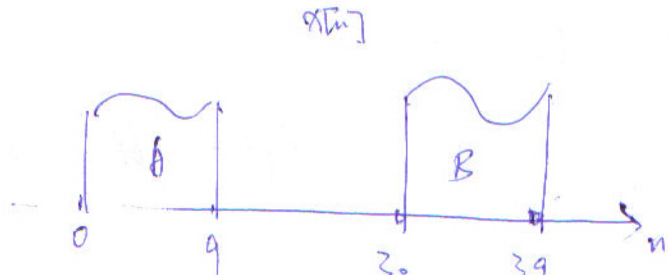
$$\Rightarrow w[0]\cdot x[-1] + w[1]\cdot x[-2] + w[2]\cdot x[-3] = 11$$

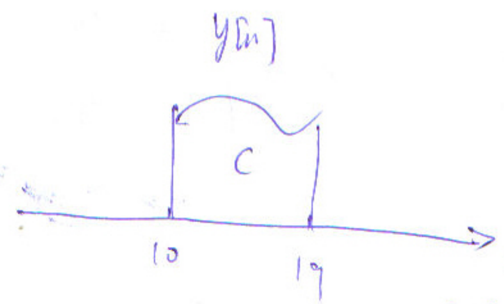
$$\Rightarrow 5 + 2\cdot\frac{A_2}{2} + 0 = 11 \Rightarrow A_2 = 6$$

So.

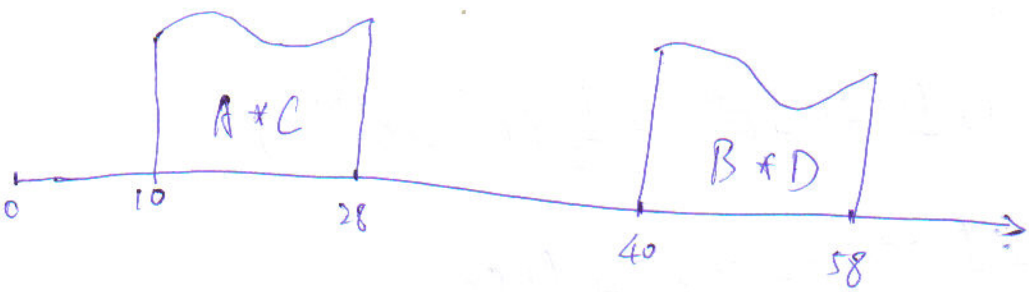
$$x[n] = \delta[n] + 5\delta[n-1] + 5\delta(n+1) + 3\delta[n-2] + 3\delta[n+2]$$

8.41





(a). linear convolution $w[n]$



(b) $w[n]$

$y[n]$ is an aliased $w[n]$ by the period of 40.

$y[n] = w[n]$ ~~$18 \le n$~~ $19 \le n \le 39$

$y[n] = w[n+40]$ $0 \le n \le 9$

8.43. (a) divide the input of 10000 points into sections of the length L

$$L + 100 - 1 = 256 \Rightarrow L = 157$$

There will be $\left[\frac{10000}{157}\right] + 1 = 64$ DFT of the sections.

There is one DFT for $h(n)$

So. 65 DFT plus 64 IDFT

(Page 586 textbook)

(b) $L = 157$

Then will be $\left[\frac{10000+99}{157}\right] + 1 = 65$ DFT

So. 66 DFT plus 65 IDFT   (page 588 textbook)

(c) $N = 256$ $\quad N/2 \cdot \log_2 N = 1024$ $\quad N \cdot \log_2 N = 2048$

overlap add

multi [DFT/IDFT] = 129 × 1024 = 132096
add [DFT/IDFT] = 129 × 2048 = 264192
multi [Filtering] = 64 × 256 = 16384
add [reconstruction] = 99 × 63 = 6237

So
multi [overall] = 148480
add [overall] = 270429

Overlap save

multi [DFT/IDFT] = 131 × 1024 = 134144
add [DFT/IDFT] = 131 × 2048 = 268288
multi [Filtering] = ~~64~~ 65 × 256 = 16640

So.
multi [overall] = 150784
add [overall] = 268288

Direct Convolution (disregarding the transient period)

multi = 10000 × 100 = 1,000,000
add = 10000 × 99 = 990,000

8.48

a) No.

~~$x[n] = 0 \quad 0 \le n \le N-1$~~

$$x[n] = \frac{1}{M}\sum_{k=0}^{M-1} X[k]\cdot W_M^{kn} \qquad 0 \le n \le M-1$$

if $M \geq N$

$$x[n] = 0 \qquad 0 \le n \le N-1$$

b) Yes, it is possible

The example is constructed by the condition that $x[n]$ is zero when time aliased up to M samples

$$x[n] = \delta[n] - \delta[n-1] \qquad N = 2$$

$$M = 1$$

$$X(e^{j\omega}) = 1 - e^{-j\omega}$$

$$X(e^{j0}) = 1 - 1 = 0$$

~~$X(e^{j}$~~