Problem Set II

8.4 8.9 8.21 8.23 8.24 8.26 8.32

✓ ✓

8.4

(a) $$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= \frac{1}{1-\alpha e^{-j\omega}}, \quad |\alpha|<1$$

(b) $$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{kn} \qquad W_N = e^{-j\frac{2\pi}{N}}$$

$$= \sum_{n=0}^{N-1} \sum_{r=0}^{\infty} \alpha^{n+rN} W_N^{kn}$$

$$\tilde{X}[k] = \sum_{r=0}^{\infty} \alpha^{rN} \left(\frac{1-\alpha^N e^{-j2\pi k}}{1-\alpha e^{-j\frac{2\pi k}{N}}} \right), \quad |\alpha|<1$$

$$= \frac{1}{1-\alpha^N} \left(\frac{1-\alpha^N e^{-j2\pi k}}{1-\alpha e^{-j\frac{2\pi k}{N}}} \right)$$

$$= \frac{1}{1-\alpha e^{-j\frac{2\pi k}{N}}}, \quad |\alpha|<1$$

(c) $$\tilde{X}[k] = X(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{N}}.$$

8.9

(a) $N=5 \quad k=2.$

$$X_1[n] = \sum_{r=-\infty}^{\infty} x[n+5r]$$

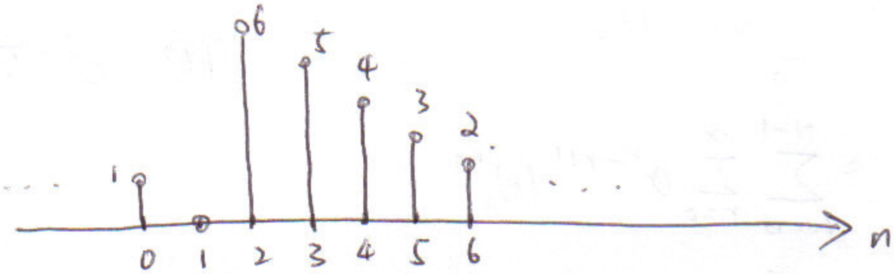
(b) $L=27 \quad k=5$

$$x_2[n] = \begin{cases} x(n) & 0 \le n \le 19 \\ 0 & 20 \le n \le 26 \end{cases}$$

9.24

$$\tilde{y}_1[n] = \sum_{m=0}^{N-1} \tilde{x}_1[m]\,\tilde{x}_2[n-m]$$

(a)



$$\tilde{y}_1[n] = \tilde{x}_1[n-2]$$

(b) $\tilde{y}_1[n] = \tilde{x}_1[n] + \tilde{x}_1[n-4]$

8.23

(a) $N > P$

$$x'(n) = \begin{cases} x[n] & 0 \le n \le P-1 \\ 0 & P \le n < N \end{cases}$$

Zero Padding

(b) $N < P$

$$x'(n) = \sum_{r=-\infty}^{\infty} x(n-rN)$$

$$\tilde{x}[n] = \frac{1}{N} \cdot \sum_{k=0}^{N-1} X[k] W_N^{-kn}.$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} x(m) \cdot e^{j\frac{2\pi}{N} \cdot k(n-m)}$$

$$= \sum_{m=-\infty}^{\infty} x(m) \cdot \left[\frac{1}{N} \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N} k(n-m)} \right]$$

$$= \sum_{m=-\infty}^{\infty} x(m) \cdot \left[\sum_{r=-\infty}^{\infty} \delta(m-n-r \cdot N) \right]$$

$$= \sum_{r=-\infty}^{\infty} x(n+rN) = \sum_{r=-\infty}^{\infty} x(n-rN).$$

8.24

No.

Counter example. $x[n] = \delta[n-1]$

$$X(e^{j\omega}) = e^{-j\omega}$$

$$X[k] = \begin{cases} 1 & k=0 \\ -1 & k=1 \end{cases} \qquad N=2.$$

$$Im\{X[k]\} = 0 \qquad k=0,1$$

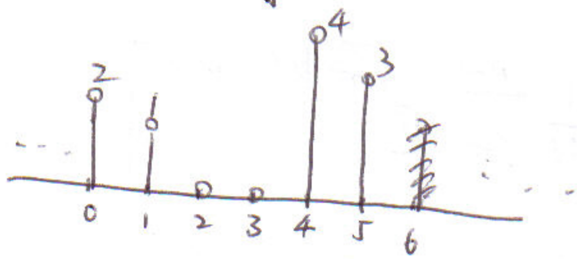
$$Im\{X(e^{j\omega})\} = -\sin\omega \qquad -\pi < \omega < \pi$$

8.26

(a)

$$y[n] = x[(n-4)_6 \;]$$

$=$



(b)

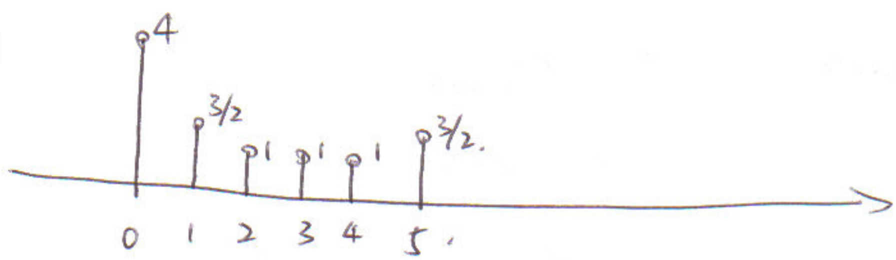
$$W[k] = \frac{1}{2}[X[k] + X^*[k]]$$

$$= \frac{1}{2}\left(4 + 3W_6^{k} + 2W_6^{2k} + W_6^{3k} + 4 + 3W_6^{-k} + 2W_6^{-2k} + W_6^{-3k}\right)$$

$$= \frac{1}{2}\left(4 + 3W_6^{k} + 2W_6^{2k} + W_6^{3k} + 4 + 3W_6^{5k} + 2W_6^{4k} + W_6^{3k}\right)$$

$$= 4 + \frac{3}{2}W_6^{k} + W_6^{2k} + W6^{3k} + W6^{4k} + \frac{3}{2}W^{5k}$$

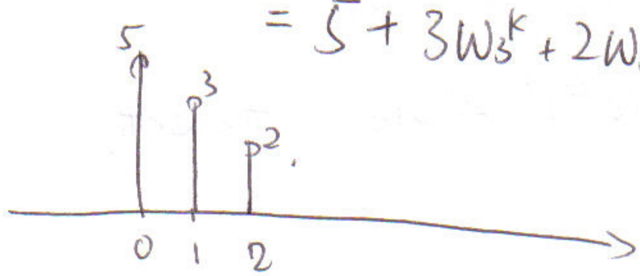
w(n)



(c)

$$Q[k] = X[2k] = 4 + 3W_6^{2k} + 2W_6^{4k} + W_6^{6k}$$

$$= 5 + 3W_3^{k} + 2W_3^{2k}.$$



8.32

The choice is C.

$$y[n] = \begin{cases} x(n/2] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$Y[k] = \sum_{n=0}^{15} y[n] W_{16}^{kn}$$

$$= \sum_{n=0}^{7} x[n] \cdot W_{16}^{2kn} = \sum_{n=0}^{7} x[n] W_8^{kn} \qquad 0 \le k \le 15.$$

That is, the 16-pt DFT of the interpolated signal contains two copies of the 8pt DFT of x[n].

Refer to Question 4.26 in Problem set 7.