Problem Set I

2.17 2.28 4.2 4.8 4.19 4.21 4.23 4.26

2.17 (a) $$R(e^{j\omega}) = \sum_{n=0}^{M} e^{-j\omega n} = e^{-j\frac{M}{2}\omega}\frac{\sin\left(\frac{M+1}{2}\omega\right)}{\sin(\omega/2)}$$

(b) $$W(e^{j\omega}) = R(e^{j\omega}) * \left[\frac{1}{2}\delta(\omega) - \frac{1}{4}\delta(\omega + \frac{2\pi}{M}) - \frac{1}{4}(\omega - \frac{2\pi}{M})\right]$$

2.28 (a) $N=5$ (c) Non periodic

(b) $N=38$ (d) Non periodic

4.2

$$\frac{\pi}{4}\cdot n = \Omega_0 \cdot \frac{1}{1000} n \Rightarrow \Omega_0 = 250\pi$$

$$\frac{\pi}{4}\cdot n + 2\pi n = \Omega_0 \frac{1}{1000} n \Rightarrow \Omega_0 = 2250\pi$$

4.8 (a) $$T \leq \frac{1}{2\times 10^4}$$

(b) $$y[n] = T\cdot\sum_{k=-\infty}^{n} x[k] = T\cdot\sum_{k=-\infty}^{\infty} x[k]\, u[n-k]$$

where $$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Since $y[n] = x[n] * h[n]$ by definition

$$h[n] = T\cdot u[n]$$

(c) $\lim_{n \to \infty} y[n] = \lim_{n \to \infty} T \cdot \sum_{n=-\infty}^{\infty} x[n] = T \cdot X(e^{j\omega})\Big|_{\omega=0}$

(d). $X(e^{j\omega}) = \frac{1}{T} \sum_{r=-\infty}^{\infty} X_c\left(\frac{j\omega}{T} + \frac{j 2\pi r}{T}\right)$

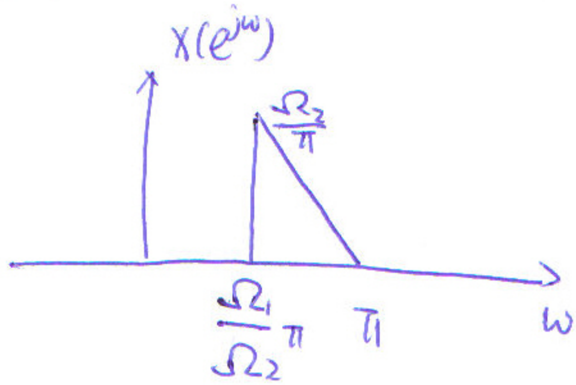
$$T X(e^{j\omega})\Big|_{\omega=0} = \sum_{r=-\infty}^{\infty} X_c\left(j \frac{2\pi r}{T}\right)$$

To avoid aliasing at DC $\omega=0$, instead of (a), there is

$$T \leq \frac{4\pi}{2 \cdot \max(|\Omega|)} = \frac{1}{1 \times 10^4}$$

4.19 $T \leq \frac{\pi}{\Omega_0}$

4.21 (a)



2 steps of scaling on spectrum

① Scaling on the magnitude by a factor of $\frac{1}{T}$

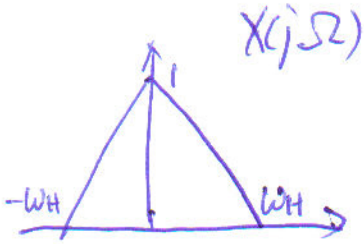
② Scaling on the frequency by a factor of $\frac{1}{T}$

(b). $T \leq \frac{2\pi}{\Delta\Omega}$ Note it is a sampling on the Bandpass signal.

(c) $x(n) \to$ impulse train $\to$ Band pass filter $\to X_c(t)$

4.23

Look what happens on the frequency Domain.



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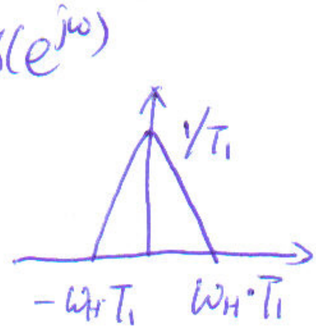

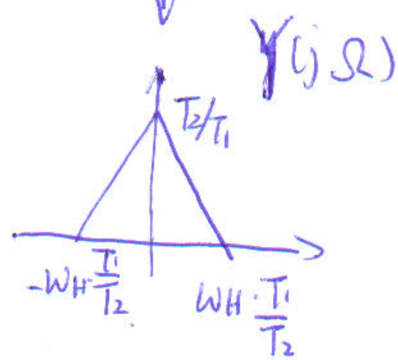

$$Y(j\Omega) = \frac{T_2}{T_1} X\left(j\frac{T_2}{T_1}\Omega\right)$$

$$\Downarrow$$

$$\boxed{y_c(t) = \frac{T_2}{T_1} x_c\left(\frac{T_2}{T_1} t\right)}$$

Reconstruction introduces no aliasing.

4.26

$$X_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} X_s[Mn] e^{-j\omega n}$$

$$= \sum_{l=-\infty}^{\infty} X_s[l] e^{-j(\omega/M) l}$$

$$= X_s(e^{j\,\omega/M})$$

$X_s(n)$ is a down sampled $X(n)$ by a factor of M

$X_d(n)$ interpolates $X_s(n)$ with zeros, by a factor of M, recovering the sampling frequency of $x(n)$

(a).

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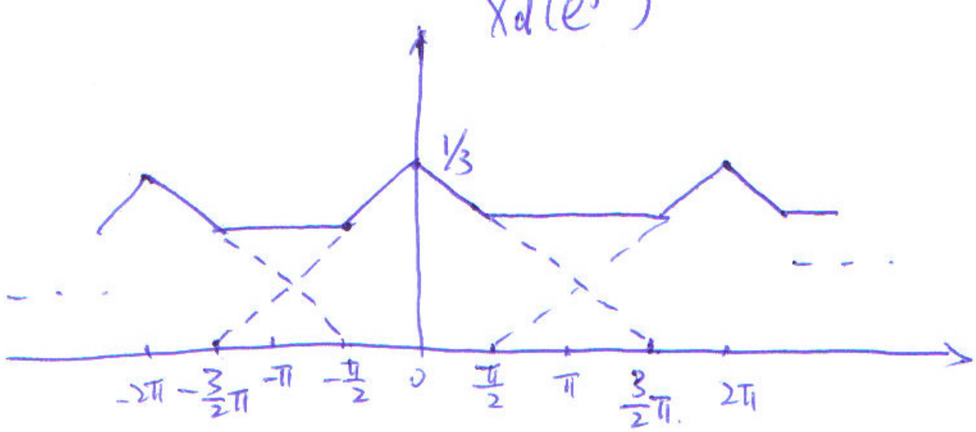

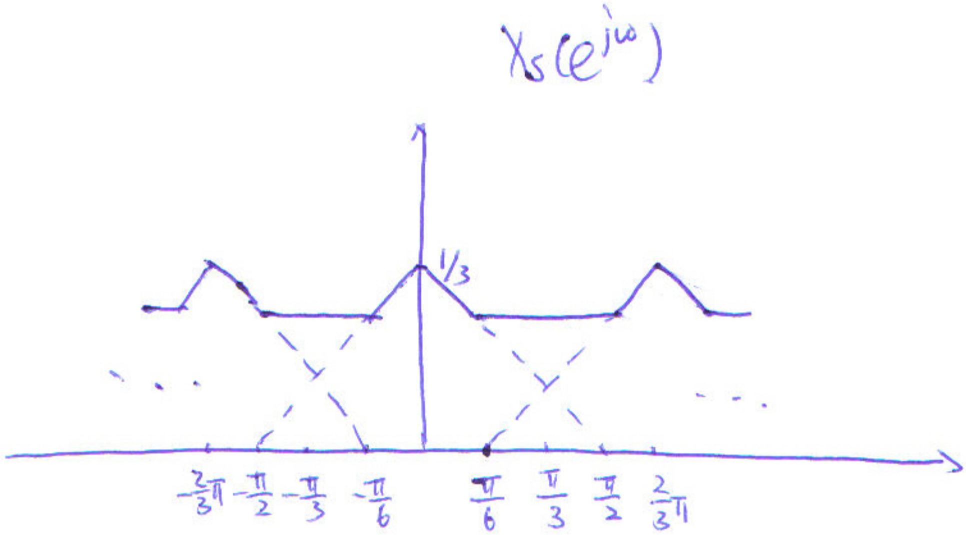

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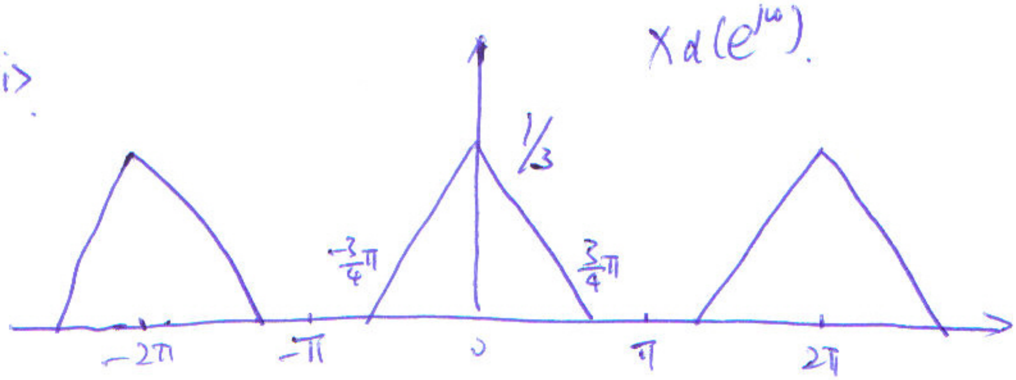

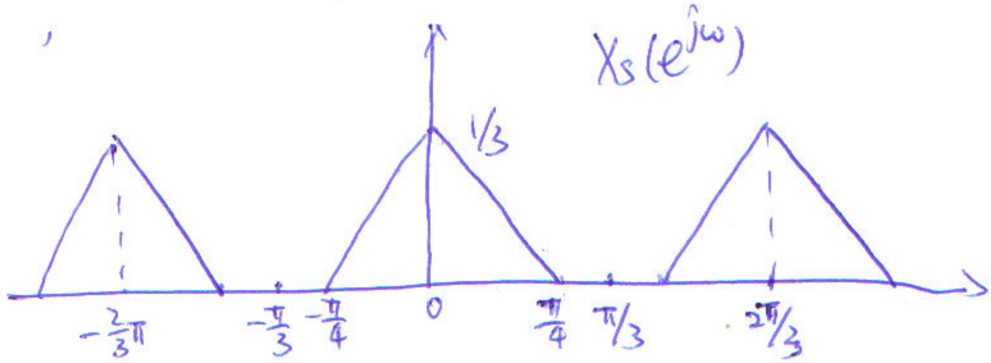

(b) $\omega_H \leq \frac{\pi}{3}$