From eqn. 1 we obtain

$$\Delta Q_0^{(1)}(z) = C_0\left[1 + \frac{C_0}{C_2(C_0 + C_1)}\frac{-C_0z^{-4} + C_1z^{-8}}{1 + \left(\frac{C_3}{C_2} - 1\right)z^{-4}}\right]V_0^{(1)}(z) \quad (3)$$

Hence, for the capacitance values

$$C_0 = 3C_1 = 4C_2 = 2C_3$$

we have

$$\Delta Q_0^{(1)}(z) = C_0\frac{(1 - z^{-4})^2}{1 + z^{-4}}V_0^{(1)}(z)$$
$$= (1 - z^{-4})Q_0^{(1)}(z) \quad (4)$$

and the total charge

$$Q_0^{(1)}(z) = \frac{C_0}{2}2\frac{1 - z^{-4}}{1 + z^{-4}}V_0^{(1)}(z) \quad (5)$$

The charge of the analogue reference FDNR is given by the relation in the s domain

$$Q_0(s) = sDV_0(s) = \frac{D}{T}sTV_0(s) \quad (6)$$

It follows from eqns. 5 and 6 that for $C_0/2 = D/T$ we obtain

$$sT = 2\frac{1 - z^{-4}}{1 + z^{-4}} \quad (7)$$

This means that the SC circuit in Fig. 1*a* simulates an FDNR according to the bilinear s–z transformation.

D is the constant factor in the FDNR with resistance $R_e = 1/s^2D$.

Omitting the capacitor C_3 ($C_3 = 0$) with the corresponding switches and changing the polarity by switching the capacitor C_0 from clock phase 3 to clock phase 4 we obtain

$$\Delta Q_0^{(1)}(z) = C_0\frac{(1 + z^{-4})^2}{1 - z^{-4}}V_0^{(1)}(z)$$
$$= (1 - z^{-4})Q_0^{(1)}(z) \quad (8)$$

and the total charge

$$Q_0^{(1)}(z) = 4C_0\frac{1}{\left(2\frac{1 - z^{-4}}{1 + z^{-4}}\right)^2}V_0^{(1)}(z) \quad (9)$$

This means that such a circuit simulates an inductor as the charge of analogue reference inductor is given by the relation in the s domain

$$Q_0(s) = \frac{1}{s^2L}V_0(s) = \frac{T^2}{L}\frac{1}{s^2T^2}V_0(s) \quad (10)$$

and from eqns. 9 and 10 for $4C_0 = T^2/L$ we again obtain

$$sT = 2\frac{1 - z^{-4}}{1 + z^{-4}}$$

The equivalent inductance of this SC circuit is given by the relation $L_e = T^2/4C_0$. Such a circuit is also presented in References 1 and 2.

A new SC FDNR controlled by four clock phases is presented. The network with only one opamp and four capacitors is very economical. This FDNR can be used effectively in solving SC filters.

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References

1 CICHOCKI, A., and UNBEHAUEN, R.: 'Simplified analysis of arbitrary switched-capacitor networks', *IEE Proc. G*, 1987, **134**, pp. 45–53

2 UNBEHAUEN, R., and CICHOCKI, A.: 'MOS switched-capacitor and continuous-time integrated circuits and systems' (Springer-Verlag, Berlin, 1989)

3 DABROVSKI, A., and MOSCHYTZ, G. S.: 'Direct by-inspection derivation of signal-flow graphs for multiphase stray-insensitive switched-capacitor filters', *Electron. Lett.*, March 1989, **25**, pp. 387–388

'CHIRPLETS' AND 'WARBLETS': NOVEL TIME–FREQUENCY METHODS

S. Mann and S. Haykin

Indexing terms: Radar, Transforms, Mathematical techniques

A novel transform is proposed, which is an expansion of an arbitrary function onto a localised basis of multiscale chirps (swept frequency wave packets) for which the term 'chirplets' has been used. The wavelet transform is an expansion onto a basis of functions which are affine in the physical domain (e.g. time). In other words they are translates and dilates of one mother wavelet. The proposed basis is an extension of affinity, from the physical (time) domain, to the time–frequency domain. The basis includes both the wavelet and the short-time Fourier transform (STFT) as special cases (the degree of freedom *modulation* is simply attained through a translation in frequency). Furthermore, the bases include shear in time, and shear in frequency, leading to a broader class of chirping bases. Numerous practical applications of the chirplet have been found, such as in Doppler radar signal processing.

The well-known wavelet transform was originally derived through one-dimensional affine transformations in the physical (e.g. time) domain. Our proposed chirplet bases, however, are derived through the six 2-D affine transformations in the time–frequency (TF) plane.

The chirplet transform thus has indexdimension up to 6† (depending on the particular 'mother chirplet' chosen), rather than 2, as is the case with the wavelet transform. Chirplet theory allows for a unified framework because it embodies many other TF methods as lower dimensional manifolds in chirplet space. For example, both the wavelet transform, and the short-time Fourier transform (STFT) are planar chirplet slices while many adaptive methods are two indexdimensional curved chirplet manifolds.

'Le pèpielette': 'The chirplet': Y. Meyer is often said to be the 'father of wavelets'; it was he* who first coined the term «ondelette» (from the French word for wave, «onde», and the diminutive «ette»). The closest translation, 'wavelet', became the accepted word within papers written in the English language.

We use the term «pépielette» (combining the diminutive with the French word «pépier» or «pèpiement») to similarly designate a 'piece of a chirp'. An English translation gives us the word 'chirplet'. Fig. 1 shows that the relationship of a chirplet to a chirp is analogous to that of a wavelet to a wave.

The first published reference‡ is in Mann and Haykin [2]; the term also appears elsewhere in the literature [4].§

We propose a basis which consists of all the members of a particular time domain signal, which are affine transformations of each other when viewed in TF space. Philosophically there are two ways to think of this basis function:

(1) using the 'piece-of-a-chirp' framework

† The high indexdimensionality may be dealt with either by examining lower dimension manifolds, or by using an adaptive algorithm [1]
* *Editor's note:* Or, according to some, A. Grossmann and J. Morlet
‡ Some of the work of Mann *et al.* [2] was presented briefly as part of 'Radar vision' [3], prior to publication
§ *Editor's note:* There is some uncertainty regarding priority of coining the term 'chirplet'. Interested readers should take into account the dates on which the papers were received

(2) thinking in terms of affine transformations in TF space, which consist of dilations and 'chirpings' (in both time and frequency); we note that translations (modulations and delays) are just special cases of 'chirpings' (in time and frequency), where the chirp rate is zero

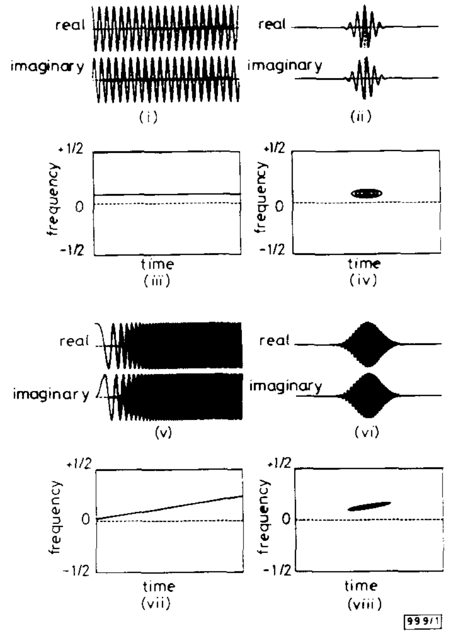

Fig. 1 *Relationship between wave, 'wavelet', chirp and 'chirplet', in terms of time series and magnitude time–frequency (TF) distributions*

We have extended the one dimensional affinity of the wavelet to two dimensions, by adding up–down translation and shear in TF space. These extra affine transformations are achieved by multiplication of the wavelet by a chirp (modulation, which is multiplication by a pure tone, is just a special case of 'chirping'). Hence we use the term 'chirplet'. The chirp in time performs a shear along the frequency axis. The nature of the Gabor function eliminates two of the six TF-affine degrees of freedom which a general chirplet has

(i) wave time series
(ii) wavelet time series
(iii) TF for wave
(iv) TF for wavelet
(v) chirp time series
(vi) chirplet time series
(vii) TF for chirp
(viii) TF for chirplet

Prolate chirplet: We illustrate our TF affine concept by a simple example. We use a function which is somewhat rectangular in TF space, the discrete prolate spheroidal sequence (DPSS). These functions are of special interest in the signal processing community (Landau, Pollack, Slepian [5, 6]) and are commonly referred to as prolates or Slepians.** When we apply our TF affine transformations to the prolate, we obtain a specific class of chirplets which we refer to as prolate chirplets.†† The chirplet has all six degrees of freedom when the prolate is chosen as the mother chirplet (shown in Fig. 2):

(1) Translation in time (may be accomplished by Fourier transformation, followed by modulation, followed by inverse Fourier transformation):

(2) Translation in frequency (modulation)

(3) Dilation in time

(4) Dilation in frequency

(5) Shear in time (chirping)

(6) Shear in frequency (Fourier transformation, followed by chirping, followed by inverse Fourier transformation).

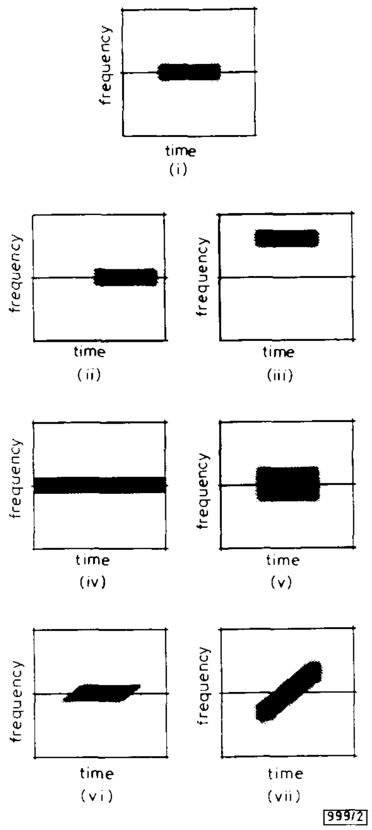

Fig. 2 *Illustration of affine transformations in TF space*

Here we use a family of discrete prolate spheroidal sequences (DPSS), and arrive at our prolate chirplet family
There are six degrees of freedom
Note that time bandwidth product is free to vary, although it has a lower bound

(i) original function
(ii) translation in time
(iii) translation in frequency
(iv) dilation in time
(v) dilation in frequency
(vi) shear in time
(vii) shear in frequency

Warblets and the winking phenomenon: Tests on actual radar data, pertaining to ocean surveillance, show that the radar return from small ice fragments rises and falls in frequency in a periodic manner [3]. (From the perspective of a surfer, ocean waves rise and fall periodically, so it stands to reason that the Doppler tone (velocity) that is obtained from a target also rises and falls periodically in pitch.)

To match this physical phenomenon we have selected a particular 'mother chirplet', to which we apply the six TF-affine operators. Because this particular choice of chirplet has a profound significance, we have given it a special name, the 'warblet'. Warblets are chirplets where the mother chirplet is a single tone FM signal (like the sound produced by either a police siren or the bird known as a warbler).

A particular manifold in warblet space, the modulation-index versus modulation-frequency plane, has been found to be very useful in analysing actual radar data, making use of

** We have applied the TF-affine operators to a number of different signals; here the Slepian is chosen simply because it is the most illustrative of the concept, not necessarily because it gives the best performance

†† We have also successfully implemented a new variant of the Thomson method of spectral estimation using a family of these prolate chirplets as multiple data windows

the winking phenomenon. A general theory has been built up around this particular planar slice.

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References

1 MANN, S., and HAYKIN, S.: 'An adaptive wavelet like transform'. SPIE, 36th Annual Int. Symp. on Optical and Optoelectronic Applied Science and Engineering, 21st–26th July 1991
2 MANN, S., and HAYKIN, S.: 'The generalized logon transform (GLT)'. Vision Interface '91, 3rd–7th June 1991
3 HAYKIN, S.: 'Radar vision' (Second International Specialist Seminar on Parallel Digital Processors, Portugal, April 1991)
4 MIHOVILOVIC, D., and BRACEWELL, R. N.: 'Adaptive chirplet representation of signals on time-frequency plane', *Electron. Lett.*, 1991, **27**, (13), pp. 1159–1161
5 SLEPIAN, D., and POLLACK, H. O.: 'Prolate spheroidal wave functions, Fourier analysis and uncertainty', *Bell Syst. Techn. J.*, January 1961, **40**, pp. 43–64
6 SLEPIAN, D.: 'Prolate spheroidal wave functions, Fourier analysis and uncertainty, V: the discrete case', *Bell Syst. Tech. J.*, May–June 1978, **57**, pp. 1371–1430

PERFORMANCE COMPARISON OF SUBCARRIER MULTIPLEXED COHERENT SYSTEMS USING OPTICAL INTENSITY AND PHASE MODULATORS

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Indexing term: Optical communications

The performance of subcarrier multiplexed coherent systems is compared theoretically using optical intensity and phase modulators. The results show that a subcarrier multiplexed coherent system with a symmetrical optical intensity modulator can offer a receiver sensitivity improvement as high as 9·1 dB over that with an optical phase modulator.

Coherent systems offer improved receiver sensitivity and frequency selectivity over direct detection systems. Subcarrier multiplexed (SCM) coherent optical systems have been studied theoretically and experimentally [1–3]. There have been reports on subcarrier multiplexed coherent systems using optical phase modulators [1]. The main factor limiting the system performance is the intermodulation distortion (IMD) due to nonlinearity of the modulation. Multioctave operation is often used to improve the bandwidth efficiency. However, in a coherent SCM system using an optical phase modulator, second order intermodulation distortion degrades the system performance seriously [1]. We compare the performance of coherent SCM systems theoretically using optical intensity and phase modulators. The results show that a coherent SCM system with a symmetrical optical intensity modulator can offer a receiver sensitivity improvement as high as 9·1 dB over that with an optical phase modulator.

Fig. 1 shows the structure of a symmetrical Mach–Zehnder intensity modulator, schematically. The optical signals passing through the two arms are phase-modulated by the electro-optic effect. The voltages applied on the two arms produce opposite optical phase shifts. By biasing the modulator at the zero-crossing point, the electrical field of the output optical signal can be written as

$$e_0(t) = \frac{E_i}{2} \cdot \{\cos[2\pi f t + \varphi_m(t)] - \cos[2\pi f t - \varphi_m(t)]\} \quad (1)$$

where E_i and f are the electrical field and the frequency of the input optical signal, respectively. $\varphi_m(t)$ is the phase modulation produced by the subcarrier signals, where

$$\varphi_m(t) = \sum_{i=1}^{N} \beta_i \cdot \sin[2\pi f_i t + \theta_i(t)] \quad (2)$$

β_i, f_i, and $\theta_i(t)$ are the optical phase modulation index, subcarrier frequency and angle modulation of the ith subcarrier, respectively.

The optical signal can be described using a Bessel function expansion [1]. This yields

$$e_0(t) = E_i \cdot \sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} \cdots \sum_{n_N=-\infty}^{+\infty} \frac{\left[1-(-1)^{\sum_{i=1}^{N} n_i}\right]}{2} \times J_{n_1}(\beta_1) \cdot J_{n_2}(\beta_2) \cdots J_{n_N}(\beta_N) \times \cos\left\{2\pi f t + \sum_{i=1}^{N} n_i \cdot [2\pi f_i t + \theta_i(t)]\right\} \quad (3)$$

where $J_n(\;)$ denotes the nth-order Bessel function of the first kind.

We find that the even-order intermodulation products are cancelled whereas the signals and the odd-order intermodulation products remain the same as that of a phase modulator.

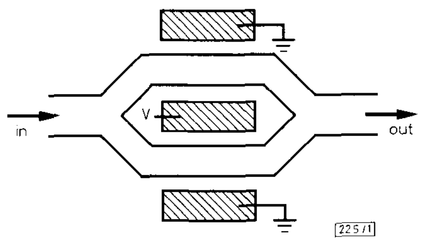

Fig. 1 *Schematic diagram of symmetrical Mach–Zehnder intensity modulator*

Based on the above analysis, we summarise our study in Tables 1 and 2. We assume a balanced receiver is used so that the direct detection terms are removed in the receiver.

Table 1 RECEIVER SENSITIVITY COMPARISON

	Multioctave	Single octave
Intensity modulator	A	A
Phase modulator	B	A

B (dB)–A (dB): receiver sensitivity improvement

(1) In a multi-octave operation, the intensity modulator results in a receiver sensitivity improvement over that of a phase modulator because of the cancellation of the second order intermodulation products. In a single-octave operation, the intensity modulator has the same performance as a phase modulator.

(2) For coherent SCM systems with 20, 30 and 40 channels in 2, 3 and 4-octave operation [1], the intensity modulator will result in a receiver sensitivity improvement of 7·5, 8·7 and 9·1 dB, respectively, over that of a phase modulator.

Table 2 RECEIVER SENSITIVITY IMPROVEMENT

Number of channels	20	30	40
Number of octaves	2	3	4
Receiver sensitivity improvement (dB)	7·5	8·7	9·1